A Recurrent Network Architecture for Nonlinear Parameter Tracking Algorithms.

D. Al-Dabass, D. Evans and S. Sivayoganathan

Dept of Computing and Mathematics
Faculty of Computing and Technology
Nottingham Trent University
Nottingham NG1 4B
david.al-dabass@ntu.ac.uk

Abstracts: A recurrent network architecture is proposed to provide continuous estimates of the higher time derivatives of a given signal trajectory in real time. These time derivatives provide the input to a non-linear function to generate estimated parameter values of an equivalent second order dynamical model. The derivation of this function is based on the input having non zero first time derivative.

INTRODUCTION

To model the behaviour of complex natural and physical systems, the authors have recently developed a number of explicit static algorithms [1] and [2] to estimate the parameters of recurrent second order models that approximate the behaviour of these complex higher order systems. These algorithms rely on the availability of the time derivatives of the trajectory. In this paper a cascaded recurrent network architecture is proposed to estimate these derivatives in successive stages online. The technique is tested successfully on a wide range of parameter tracking algorithms ranging from the constant parameter algorithm that only requires derivatives up to order 4 to an algorithm that tracks two variable parameters and requires up to the 8th time derivatives.

PROBLEM FORMULATION

Definitions

Many physical problems are described by the solution of an initial value problem of the form:

$$\omega^{-2}.x''+2.\zeta\,.\,\omega^{-1}.\,x'+x=u$$
$$x(0)=x_0, \quad x'(0)=x'_0$$

Where ω is the natural frequency, ζ is the damping ratio, u is the input and x is the output of the system. The three parameters may be constants, variables or variable with dynamical behaviour.

Algorithm For Constant Parameters From Single Point Data

This was termed algorithm 3 in reference 1.

Consider using the 1st to 4th time derivatives at a single point. Given the second order system:

$$\omega^{-2}\,x'' + 2.\,\zeta.\omega^{-1}.x' + x = u \qquad (1)$$

Differentiate with respect to t: $\omega^{-2}\,x''' + 2.\,\zeta.\omega^{-1}.x'' + x' = 0$ (2)

divide by x'': $\omega^{-2}\,x'''/\,x'' + 2.\,\zeta.\omega^{-1} + x'/\,x'' = 0$ (3)

and differentiate with respect to t again to give:

$$\omega^{-2}.[(x''.\,x'''' - x'''^2)\,/\,x''^2] + 0 + [(x''^2 - x'.\,x''')\,/\,x''^2] = 0 \qquad (4)$$

We get expressions for estimated ω, estimated ζ, using (2), and estimated u:

$$E\omega^2 = [x''. x'''' - x'''^2] / [x'. x''' - x''^2] \qquad (5)$$
$$E\zeta = -[E\omega^{-2}\, x''' + x'] / [2. E\omega^{-1}.x''] \qquad (6)$$
$$Eu = E\omega^{-2}. x'' + 2. E\zeta. E\omega^{-1} . x' + x \qquad (7)$$

HIGH ORDER ALGORITHMS

We assume that the first and higher time derivative of u to be non 0. For simplicity we still assume that both a and b (the coefficients of x'' and x' to make symbol manipulation easier) to be constant and hence disappear on first differentiation. The extra information needed for u', u'', u''' and u'''' to be non zero is extracted from the 5th, 6th, 7th and 8th time derivatives of the trajectory. Only the case for the u' is shown here, the others for u'' etc are simple extensions of the idea and are left as an exercise for the reader.

$$a.x'' + b.x' + x = u \qquad (1)$$

Differentiate wrt to t and assume u' is non zero to give: $a.x''' + b.x'' + x' = u'$ (8)
Differentiate again and set u'' = 0 gives: $a.x'''' + b.x''' + x'' = 0$ (9)

Divide Equation 4 by x''' to isolate b: $a.x''''/x''' + b + x''/x''' = 0$ (10)
Differentiate again to eliminate b: $a.(x'''''.x'' - x''''^2)/x'''^2 + (x'''^2 - x'' . x'''')/x'''^2 = 0$ (11)

Re-arranging for a gives: $a = (x'' . x'''' - x'''^2)/(x''''' . x''' - x''''^2)$ (12)

Solve for *b* by substituting a from equ. 12 into equ. 10: $b = -x''/x''' - a.x''''/x'''$

which after substituting for a and manipulating gives: $b = (x''.x''''' - x'''.x'''')/(x''''^2 - x''' . x''''')$ (13)
We can now substitute these values for *a* and *b* into Equation 1to solve for u,
$u = a.x'' + b.x' + x$

A RECURRENT ARCHITECTURE TO ESTIMATE TIME DERIVATIVES

The structure of each cell of the recurrent network is shown in Figure 1. The output of each cell feeds the input to the next one to generate the next higher order tiome derivative. The output of the system and the cascade of 1st order recurrent network filters were simulated using the 4th order Runge-Kutta method in Mathcad. The derivatives vector is shown in Figure 2.

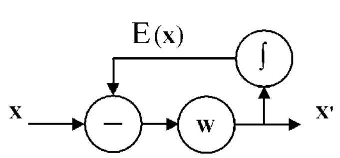

Figure 1: A single stage recurrent sub-net using an integrator in the feedback path to abduct the derivative x' = w(x-E(x))

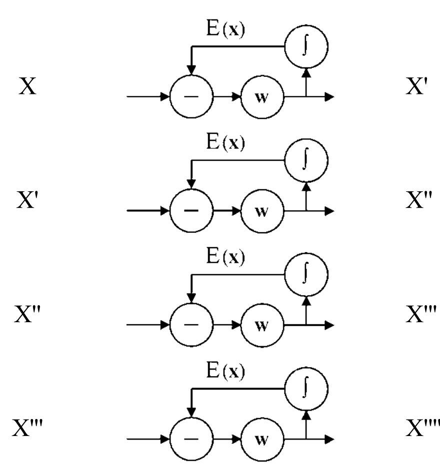

Figure 2: A 4th order recurrent network to abduct 1st to 4th time derivatives.

$$x := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad D(t,x) := \begin{bmatrix} x_2 \\ -\omega^2 \cdot x_1 - 2\cdot\zeta\cdot\omega\cdot x_2 + \omega^2\cdot u \\ G\cdot(x_1 - x_3) \\ G\cdot\left[G\cdot(x_1 - x_3) - x_4\right] \\ G\cdot\left[G\cdot\left[G\cdot(x_1 - x_3) - x_4\right] - x_5\right] \\ G\cdot\left[G\cdot\left[G\cdot\left[G\cdot(x_1 - x_3) - x_4\right] - x_5\right] - x_6\right] \\ G\cdot\left[G\cdot\left[G\cdot\left[G\cdot\left[G\cdot(x_1 - x_3) - x_4\right] - x_5\right] - x_6\right] - x_7\right] \end{bmatrix}$$

$$Z := \mathrm{Rkadapt}(x, t0, t1, N, D)$$

Figure 3: A cascade of 5 recurrent cells plus the 2nd order trajectory model.

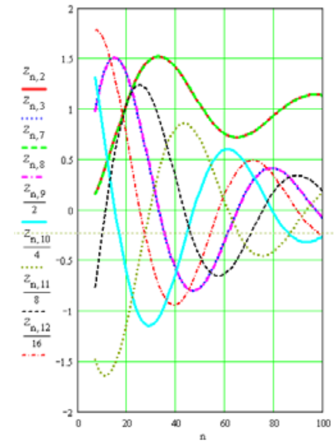

Figure 4: A typical set of time derivatives estimated from the trajectory of an oscillatory 2nd order dynamical system

RESULTS AND DISCUSSION

Results For The New Algorithm

Mathcad routines were set up to generate the input u as second order system with its own parameters of natural frequency, damping ratio and input. The input subsystem damping ratio was set to 0.05 to generate an oscillatory behaviour for long enough to test the parameter tracking algorithm thoroughly. The frequency of the input was set to 16 radians per second, one quarter of the frequency of the object natural frequency. The derivative generation cascade was increased by one to produce the fifth time derivative. The results are shown in Figure 6 below.

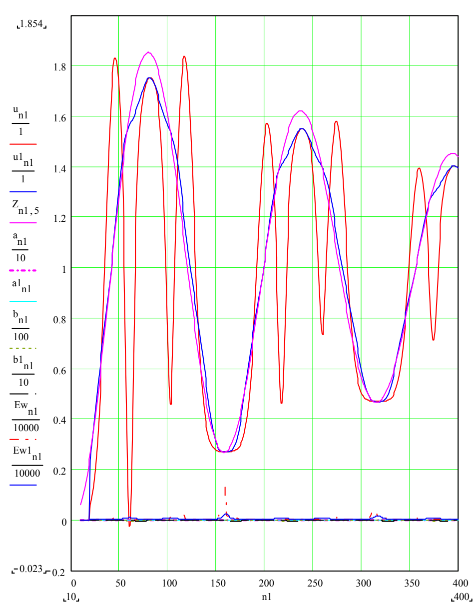

Figure 6: Results of the high order algorithm for one second integration time.

The actual input is shown in pink, which gives approximately two and a half cycles over a period of one second as expected, i.e. 16 radians/s = 2.546 Hertz. The red trace shows the results from the previous constant u derivation algorithm which is failing completely to track the input parameter. The blue trace shows the result of the new algorithm, which is managing to track the input much more closely; however it start to diverge slightly near the peak of the cycle but then returns to track it well right down and round the lower trough of the input trajectory. It is clear that tracking remain stable. It is interesting to note that the old algorithm while completely failing to track the upper half of the input trajectory it seems to track it well during the its lower half but not as well as the new algorithm.

CONCLUSIONS AND FUTURE WORK

A recurrent architecture to estimate the high order derivatives from the time trajectory of a system was proposed. These time derivatives were then used to derive and test a new high order algorithm to track the input parameter for a reduced order model. The test involved the generation of the output trajectory of a lightly damped second order system. The results showed the algorithm maintaining good tracking over an extended period of time. The new algorithm was far superior to a previous one, which relied on the assumption of constant input in the derivation.

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BIOGRAPHY: David Al-Dabass graduated from Imperial College in 1966 with BSc in Electrical Engineering, worked for Redifon Flight Simulation until 1972, completed a PhD in Parallel Processing at Staffordshire University in 1975. Between 1976 and 1982 he held post-doctoral and advanced research fellowships at the Control Systems Centre, University of Manchester Institute of Science & Technology (UMIST). He joined The Nottingham Trent University in 1983 as a Principal Lecturer in the Department of Computing and mathematics. For more details see his website:
http://ducati.doc.ntu.ac.uk/uksim/dad/webpage.htm