ESTIMATION OF SIGNAL PARAMETERS USING A NEW RECURRENT HYBRID ALGORITHM

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ABSTRACTS
A special sixth order dynamical model is proposed to simulate the behaviour of complex signals. The model consists of a two layer hierarchy of second order dynamics two of whose parameters are themselves second order. Given the trajectory of the actual complex signal, a recurrent hybrid algorithm is derived to estimate the parameters of the model. Results show good performance of the algorithm in tracking the model parameters online. Suggestions for future directions are given.

KEYWORDS
Hybrid recurrent networks, parameter estimation.

1. Introduction

Several parameter estimation algorithms have been derived, Ref 1, 2, 3. These combine estimates of a given trajectory time derivatives, using data from several points on the trajectory, with explicit static non-linear functions to provide continuous parameter estimation in real time. For time varying parameters, the time separation between the points on the trajectory directly influences the estimation accuracy. This due to the assumption of constant parameters used in the derivation is no longer valid, and accuracy deteriorates with increasing rate of parameter variation. This is termed the separation effect.

This effect can only be eliminated if all the data needed for estimation is obtained from a single time point o the trajectory. An algorithm was derived and proved, as expected, to be the most successful in coping with high rates of parameter variation. Accurate tracking of parameters when two of the parameters were varying simultaneously still proved difficult. The constant parameters assumption in the derivation is seen as the fundamental cause here.

In this paper we illustrate the use of a new algorithm which assumes the parameters to have linear time variation, with non-zero first derivative and zero second and higher time derivatives of parameters. More data is needed for this new case, which is obtained from the signal trajectory by extracting one further, higher, time derivatives. Estimation functions of the parameters are tested by generating a synthetic 6th order signal first. This is then passing it through a cascade of first order filters to estimate the time derivatives, which are finally used to track the parameters. The problem is recast in terms of recurrent hybrid networks, for both the signal model that generates the trajectory and the parameter tracking algorithm.

2. Sixth Order Models Of Complex Signals

Many physical, economical and biological phenomena exhibit a complicated behaviour even when the input parameters are constant. If the signal is modelled as a simple second order system, then to account for the complicated trajectory the input parameters themselves may be modelled as the output of other second order systems.

Consider the well known second order dynamical system which has the following form:

$$\omega^{-2}\, x'' + 2.\, \zeta.\omega^{-1}.x' + x = u$$

Where x is the output and ω, ζ and u are the natural frequency, damping ratio and input respectively, which represent the 3 parameters that form the input. To configure this differential model as a recurrent network, a twin integral elements are used to form a hybrid integral-recurrent net as shown in Fig. 1.

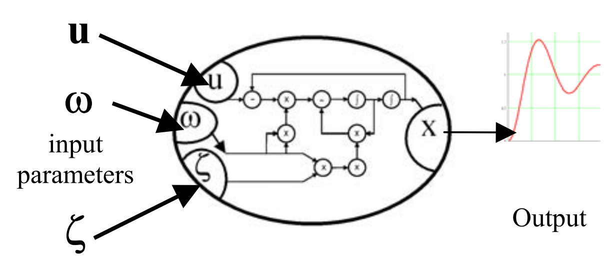


Fig. 1. Hybrid integral-recurrent net model of a 2nd order system.

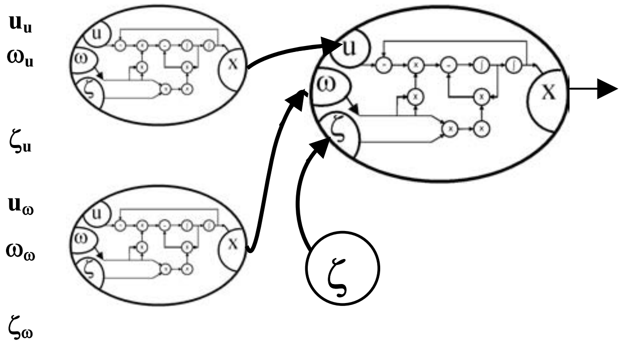


Fig. 2. Two of the input parameters of the signal are time varying and modelled with 2nd order dynamics.

2.1 Hierarchical Second Order Models

The signal trajectory is more complex than can be represented by a simple second order differential model. In this case each parameter may itself be modelled as having dynamics, which may or may not be oscillatory. One such case is where two of the 3 parameters have 2nd order characteristics, as shown in Fig. 2.

The 2nd order model of a node in a given layer in the hierarchy is given by:

$$\omega^{-2}\, x'' + 2.\, \zeta.\omega^{-1}.x' + x = u$$

To model complicated signals let both u and omega have their own 2nd order dynamics. The input **u** is the output of the following 2nd order system:

$$\omega_u^{-2}\, \mathbf{u''} + 2.\, \zeta_u\, .\, \omega_u^{-1}.\mathbf{u'} + \mathbf{u} = \mathbf{u_u}$$

The natural frequency **ω** is the output of the following 2nd order system:

$$\omega_\omega^{-2}\, \boldsymbol{\omega''} + 2.\, \zeta_\omega\, .\, \omega_\omega^{-1}.\boldsymbol{\omega'} + \boldsymbol{\omega} = \mathbf{u}_{\boldsymbol{\omega}}$$

Thus the behaviour trajectory is generated by the following 6th order vector differential equation (using Runge Kutta in Mathcad for this example), see Fig. 3.

Where x1 and x2 represent the x and x', x3 and x4 represent u and u', and x5 and x6 represent ω and ω' respectively. To generate the trajectory shown in Fig. 5, the following values were used: for the u subsystem, u started from 0 aiming at uu=1 at a rate of $\omega_u = 5$ rad/s with $\zeta_u = 0.3$. For the ω subsystem, ω started from 4 rad/s aiming at $u_\omega = 32$ rad/s at a rate of $\omega_\omega = 4$ rad/s with $\zeta_\omega = 0.1$. The resulting compound trajectory of x (red), together with the trajectories for u (blue dotted) and ω (green dotted) are shown in the graph below.

$$D(t,x) := \begin{bmatrix} x_2 \\ x_5 \cdot x_5 \cdot x_3 - 2 \cdot z \cdot x_5 \cdot x_2 - x_5 \cdot x_5 \cdot x_1 \\ x_4 \\ (wu \cdot wu \cdot uu) - 2 \cdot zu \cdot wu \cdot x_4 - wu \cdot wu \cdot x_3 \\ x_6 \\ (ww \cdot ww \cdot uw) - 2 \cdot zw \cdot ww \cdot x_6 - ww \cdot ww \cdot x_5 \end{bmatrix}$$

Fig. 3. Simulation vector of a 6th order trajectory (top 2 rows) with u (rows 3 and 4) and omega (rows 5 and 6) of the signal having 2nd order dynamics.

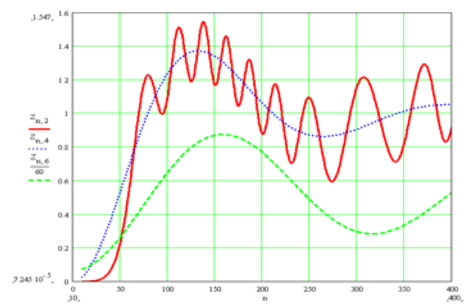


Fig. 4. Simulated trajectory of a complex signal of a sixth (red trace), with 2 variable inputs: u (blue) and omega (green).

3. Parameter Tracking Algorithms

There are several algorithms for estimating the parameters of signal generating models, see ref 1. A reduced order model that can be used to describe the oscillatory behaviour of systems forms an initial value problem of the type:

$$\omega^{-2}.x''+2.\zeta\, .\, \omega^{-1}.\, x'+x=u \quad x(0)=x_0, \quad x'(0)=x'_0$$

Where ω is the natural frequency, ζ is the damping ratio, u is the input and x is the output of the system. The three parameters may be constants, variables or variable with dynamical behaviour.

3.1 Models Assuming Constant Parameters From Single Point Data

Consider using the 1st to 4th time derivatives at a single point. Given the second order system:

$$\omega^{-2}\, x'' + 2.\, \zeta.\omega^{-1}.x' + x = u \tag{1}$$

Differentiate with respect to t:

$$\omega^{-2} x''' + 2.\ \zeta.\omega^{-1}.x'' + x' = 0 \quad (2)$$

Divide by x'':

$$\omega^{-2} x'''/ x'' + 2.\ \zeta.\omega^{-1} + x'/ x'' = 0 \quad (3)$$

Differentiate with respect to t again to give:

$$\omega^{-2}.[(x''.\ x'''' - x'''^2) / x''^2] + 0 + [(x''^2 - x'.\ x''') / x''^2] = 0 \quad (4)$$

We get expressions for estimated ω, estimated ζ, using (2), and estimated u:

$$E\omega^2 = [x''.\ x'''' - x'''^2] / [x'.\ x''' - x''^2] \quad (5)$$
$$E\zeta = -[E\omega^{-2} x''' + x'] / [2.\ E\omega^{-1}.x''] \quad (6)$$
$$Eu = E\omega^{-2}.\ x'' + 2.\ E\zeta.\ E\omega^{-1}.\ x' + x \quad (7)$$

3.2 Models for Time Varying Parameters

We assume that the first time derivative of u to be non 0. For simplicity we still assume that both a and b (the coefficients of x'' and x' to make symbol manipulation easier) to be constant and hence disappear on first differentiation. The extra information needed for u' to be non zero is extracted from the 5th time derivative of the trajectory.

$$a.x'' + b.x' + x = u \quad (1\text{-a})$$

Differentiate wrt to t and assume u' is non zero to give:

$$a.x''' + b.x'' + x' = u' \quad (8)$$

Differentiate again and set u'' = 0 gives:

$$a.x'''' + b.x''' + x'' = 0 \quad (9)$$

Divide Equation 4 by x''' to isolate b:

$$a.x''''/x''' + b + x''/x''' = 0 \quad (10)$$

Differentiate again to eliminate b:

$$a.(x'''''.x'' - x''''^2)/x'''^2 + (x'''^2 - x''.\ x'''')/x'''^2 = 0 \quad (11)$$

Re-arranging for a gives:

$$a = (x''.\ x'''' - x'''^2)/(x'''''.\ x''' - x''''^2) \quad (12)$$

Solve for *b* by substituting a from equ. 12 into equ. 10:

$$b = -x''/x''' - a.x''''/x'''$$

which after substituting for a and manipulating gives:

$$b = (x''.x''''' - x'''.x'''')/(x''''^2 - x'''.\ x''''') \quad (13)$$

We can now substitute these values for *a* and *b* into Equation 1to solve for u,

$$u = a.x'' + b.x' + x$$

The essence of the algorithm relies on the accurate estimation of the time derivatives online/real time as the trajectory is tracked This is the subject of the next section.

4. Trajectory Time Derivatives Estimation

The time derivatives of the trajectory are estimated using a cascade of 1st order recurrent networks, Fig. 5. The output of each cell feeds the input to the next one to generate the next higher order time derivative. The output of the system and the cascade of 1st order recurrent network filters were simulated using the 4th order Runge-Kutta method in Mathcad. The derivatives vector for producing derivatives up to fifth is shown in Fig. 6. Figure 7 shows a typical set of high order time derivatives estimated from the signal.

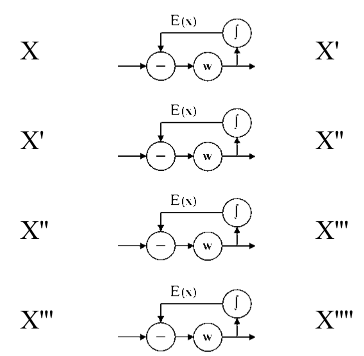


Fig. 5. A 4th order recurrent network to estimate 1st to 4th time derivatives.

$$D(t,x) := \begin{bmatrix} x_2 \\ x_5 \cdot x_5 \cdot x_3 - 2 \cdot z \cdot x_5 \cdot x_2 - x_5 \cdot x_5 \cdot x_1 \\ x_4 \\ (wu \cdot wu \cdot uu) - 2 \cdot zu \cdot wu \cdot x_4 - wu \cdot wu \cdot x_3 \\ x_6 \\ (ww \cdot ww \cdot uw) - 2 \cdot zw \cdot ww \cdot x_6 - ww \cdot ww \cdot x_5 \\ G \cdot (x_1 - x_7) \\ G1 \cdot [G \cdot (x_1 - x_7) - x_8] \\ G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] \\ G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] - x_{10}] \\ G4 \cdot [G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] - x_{10}] - x_{11}] \\ G5 \cdot [G4 \cdot [G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] - x_{10}] - x_{11}] - x_{12}] \end{bmatrix}$$

Fig. 6 The simulation derivative vector, the signal output (top 2 rows), with the input u modelled in rows 3 and 4, and omega in rows 5 an 6; rows 7 to 12 estimate the derivatives as 1st order recurrent nodes.

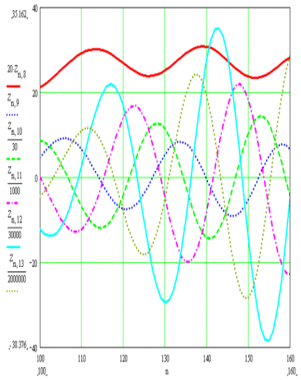


Fig. 7. A typical signal trajectory (red) and 5 higher order time derivatives x' to x''''' estimated from it.

5. Results And Discussion

Mathcad routines were set up to generate the input u as second order system with its own parameters of natural frequency, damping ratio and input. The input subsystem damping ratio was set to 0.05 to generate an oscillatory behaviour for long enough to test the parameter tracking algorithm thoroughly. The frequency of the input was set to 10 radians per second; the frequency of the main signal, on the other hand, started from 20 and aimed at 80 with a peak of about 130 radians. The derivative generation cascade was increased by one to produce the fifth time derivative. The results are shown in Fig. 8 to 10 below.

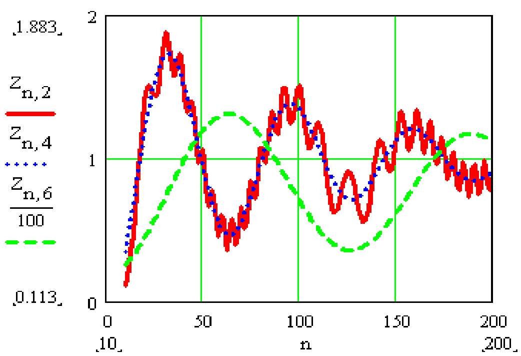


Fig. 8. The signal (red) following a damped 2nd order input (blue dotted) with variable natural frequency (green dotted); U: omega=10 rad/s, zeta=0.1, input=1 starting from 0; Omega: omega=5 rad/s, zeta=0.05, input=80 starting from 20 rad/s.

Tracking Two Parameters: For a given range of parameters the algorithm worked well, being able to estimate the two input parameters u and omega with their

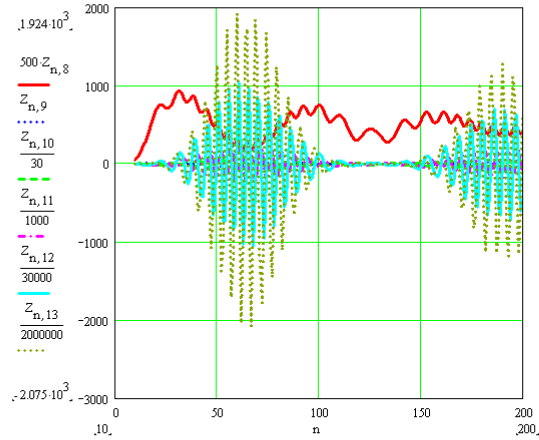


Fig. 9. The signal (red) and its estimated high order derivatives : high values during periods of high frequency followed by low valued for low frequency

time varying behaviour, i.e. track them while they are changing.

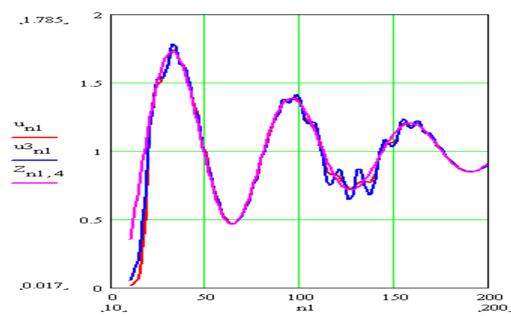


Fig 10. Deterioration in the accuracy of estimated input u during periods of low value high derivatives, particularly prominent for the high order estimation algorithm, e.g. 3rd order shown in blue against 1st order (red) and actual value (pink).

The 1st order algorithm worked better than the constant one, Fig. 11 shows a comparison of the two algorithms tracking omega.

Fig. 11. Estimated Omega assuming constant parameters (Ew, red tarce) and first order parameters (Ew1, blue trace) compared to actual shown in pink

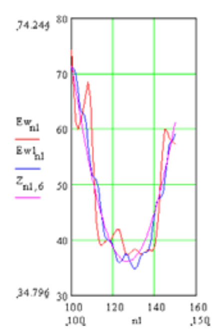
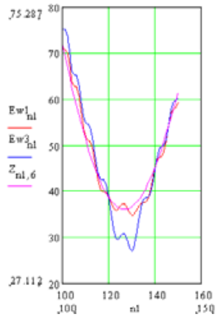


Fig. 12. Deterioration of accuracy of estimated Omega with higher order algorithms: 3rd order estimate (Ew3 in blue) is worse than 1st order estimate (Ew1 in red), actual in pink.



Algorithms of higher order than 1st showed marginal improvement but in certain cases showed a deteriorating behaviour, Fig. 12 shows a 3rd order algorithm deviating quite markedly from the true trajectory compared to a 1st order algorithm. This is likely to be due to an accumulation of errors in higher derivative values used in the former algorithm.

6. Conclusions And Future Work

Estimation models were derived to track the parameters of complicated signals in real time. To estimate the values of the parameters, the values of higher order time derivatives had to be estimated first. A cascade of 1st order recurrent networks were used to for this, followed by non-linear functions to track the parameter values. Several of these were tested, the one based on a 1st order parameters proved to be the most accurate overall. Higher order algorithms gave marginally more accurate results but their accuracy deteriorated under certain conditions. Future work will extend the technique to a further layer to track the parameters of a third level input model. Application to online stability estimation of large delicate structures will be explored.

BIOGRAPHY: David Al-Dabass graduated from Imperial College in 1966 with BSc in Electrical Engineering, worked for Redifon Flight Simulation until 1972, completed a PhD in Parallel Processing at Staffordshire University in 1975 and held post-doctoral and advanced research fellowships (76-82) at the Control Systems Centre, UMIST. He joined The Nottingham Trent University in 1983 as a Principal Lecturer in the Department of Computing. For more details see his website: http://ducati.doc.ntu.ac.uk/uksim/dad/webpage.htm

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