

# SIGNAL PROCESSING USING HYBRID RECURRENT MODELS FOR DATA MINING KNOWLEDGE DISCOVERY IN FINANCIAL TRAJECTORIES

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**Abstracts:** Starting with a model of the financial trajectory to be mined, a recurrent hybrid algorithm is derived to discover the knowledge embedded within the data. Results show good performance of the algorithm in discovering the data model parameters online. Suggestions for future directions are given

**Keywords:** data mining, financial trajectories, hybrid recurrent nets.

**1. INTRODUCTION**

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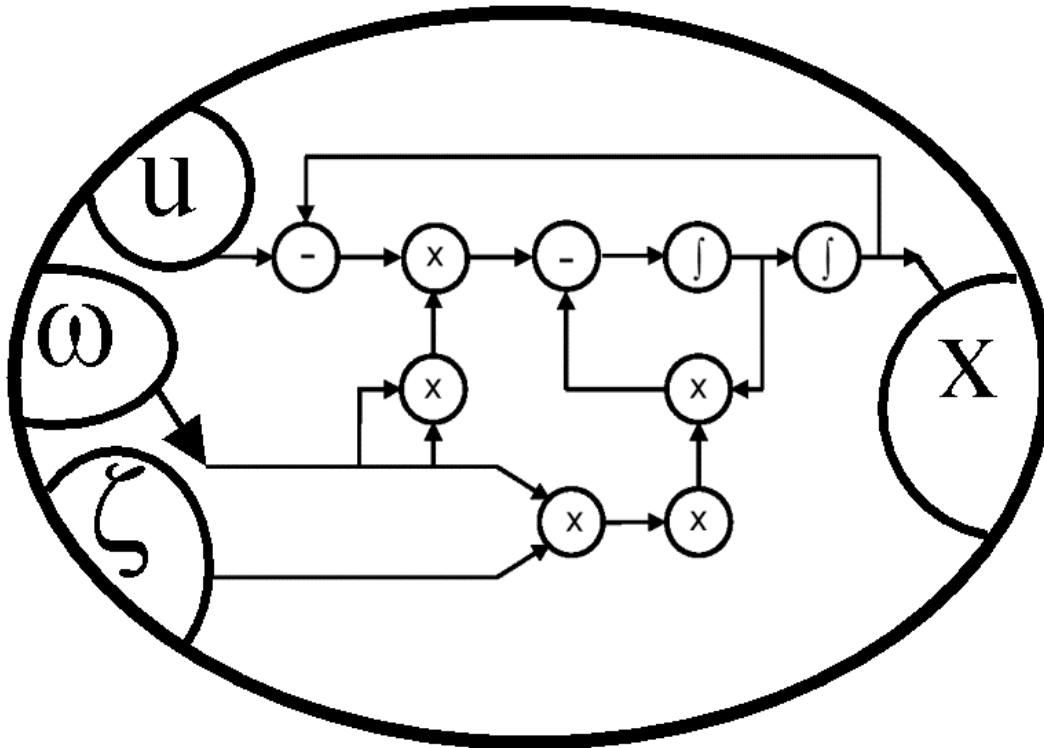
# 1. INTRODUCTION

- Several parameter estimation algorithms have been derived, Ref 1, 2, 3.
- These combine estimates of a given trajectory time derivatives, using data from several points on the trajectory, with explicit static non-linear functions to provide continuous parameter estimation in real time.
- For time varying parameters, the time separation between the points on the trajectory directly influences the estimation accuracy.
- This is due to the fact that the assumption of constant parameters used in the derivation is no longer valid, and accuracy deteriorates with increasing rate of parameter variation. This is termed the separation effect.
- This effect can only be eliminated if all the data needed for estimation is obtained from a single time point on the trajectory.
- An algorithm was derived and proved, as expected, to be the most successful in coping with high rates of parameter variation.

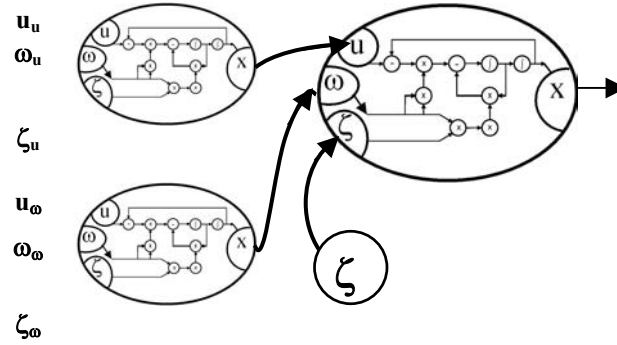
- Accurate tracking of parameters when two of the parameters were varying simultaneously still proved difficult. The constant parameters assumption in the derivation is seen as the fundamental cause here.
- In this paper we illustrate the use of a new algorithm which assumes the parameters to have linear time variation, with non-zero first derivative and zero second and higher time derivatives of parameters.
- More data is needed for this new case, which is obtained from the signal trajectory by extracting one further, higher, time derivatives.
- Estimation functions of the parameters are tested by generating a synthetic 6<sup>th</sup> order signal first.
- This is then passed through a cascade of first order filters to estimate the time derivatives, which are finally used to track the parameters.
- The problem is recast in terms of recurrent hybrid networks, for both the signal model that generates the trajectory and the parameter tracking algorithm.

## 2. HYBRID RECURRENT MODELS OF FINANCIAL TRAJECTORIES

- Many physical, economical and biological phenomena exhibit a complicated behaviour even when the input parameters are constant.
- If the signal is modelled as a simple second order system, then to account for the complicated trajectory the input parameters themselves may be modelled as the output of other second order systems.
- $\omega^{-2} \mathbf{x}'' + 2. \zeta. \omega^{-1} . \mathbf{x}' + \mathbf{x} = \mathbf{u}$
- Where  $\mathbf{x}$  is the output and  $\omega$ ,  $\zeta$  and  $\mathbf{u}$  are the natural frequency, damping ratio and input respectively, which represent the 3 parameters that form the input.
- To configure this differential model as a recurrent network, a twin integral elements are used to form a hybrid integral-recurrent net as shown in Fig. 1.



**Fig. 1.** Hybrid integral-recurrent net model of a 2<sup>nd</sup> order system.



**Fig. 2.** Two of the input parameters of the signal are time varying and modelled with 2<sup>nd</sup> order dynamics.

## 2.1 Hierarchical Second Order Models

The signal trajectory is more complex than can be represented by a simple second order differential model. In this case each parameter may itself be modelled as having dynamics, which may or may not be oscillatory. One such case is where two of the 3 parameters have 2<sup>nd</sup> order characteristics, as shown in Fig. 2.

The 2<sup>nd</sup> order model of a node in a given layer in the hierarchy is given by:

$$\omega^{-2} x'' + 2. \zeta. \omega^{-1} .x' + x = u$$

To model complicated signals let both  $u$  and  $\omega$  have their own 2<sup>nd</sup> order dynamics. The input  $u$  is the output of the following 2<sup>nd</sup> order system:

$$\omega_u^{-2} \mathbf{u}'' + 2. \zeta_u . \omega_u^{-1} .\mathbf{u}' + \mathbf{u} = u_u$$

The natural frequency  $\omega$  is the output of the following 2<sup>nd</sup> order system:

$$\omega_\omega^{-2} \omega'' + 2. \zeta_\omega . \omega_\omega^{-1} .\omega' + \omega = u_\omega$$

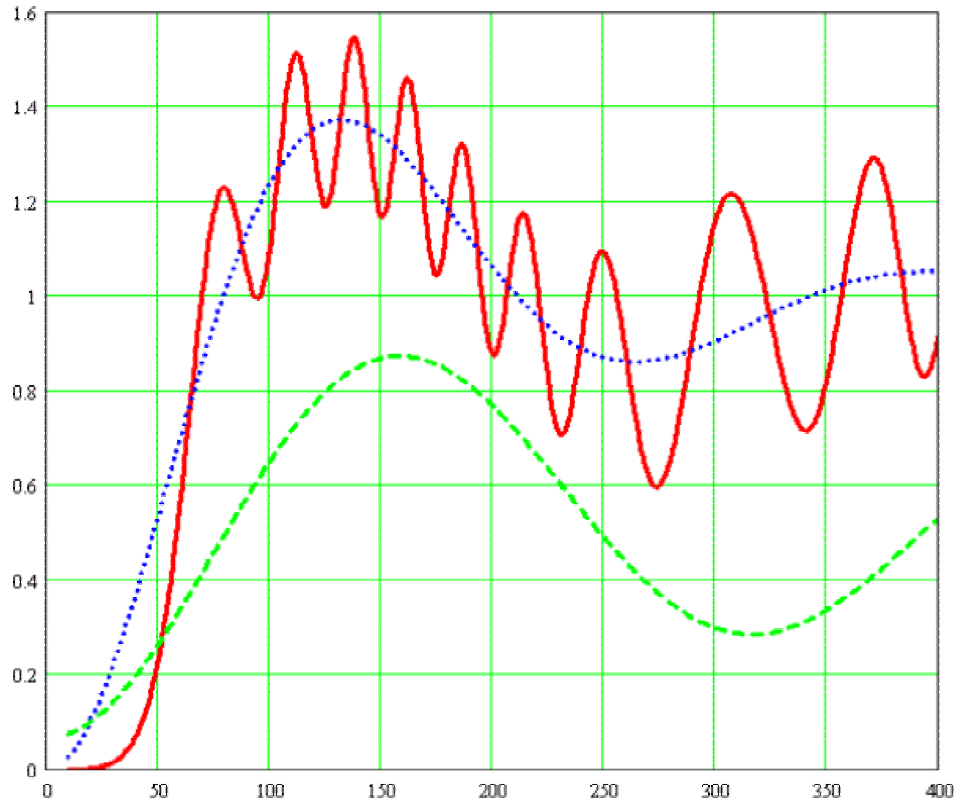
Thus the behaviour trajectory is generated by the following 6<sup>th</sup> order vector differential equation (using Runge Kutta in Mathcad for this example), see Fig. 3.

Where  $x_1$  and  $x_2$  represent the  $x$  and  $x'$ ,  $x_3$  and  $x_4$  represent  $u$  and  $u'$ , and  $x_5$  and  $x_6$  represent  $\omega$  and  $\omega'$  respectively. To generate the trajectory shown in Fig. 5, the following values were used: for the  $u$  subsystem,  $u$  started from 0 aiming at  $u_u=1$  at a rate of  $\omega_u = 5$  rad/s with  $\zeta_u = 0.3$ . For the  $\omega$  subsystem,  $\omega$  started from 4 rad/s aiming at  $u_\omega = 32$  rad/s at a rate of  $\omega_\omega = 4$  rad/s with  $\zeta_\omega = 0.1$ . The resulting compound trajectory of  $x$  (red), together

$$D(t, x) := \begin{bmatrix} x_2 \\ x_5' x_5' x_3 - 2 \cdot z x_5' x_2 - x_5' x_5' x_1 \\ x_4 \\ (w u w u u) - 2 \cdot z u w u x_4 - w u w u x_3 \\ x_6 \\ (w w w w u w) - 2 \cdot z w w w x_6 - w w w w x_5 \end{bmatrix}$$

**Fig. 3.** Simulation vector of a 6<sup>th</sup> order trajectory (top 2 rows) with  $u$  (rows 3 and 4) and  $\omega$  (rows 5 and 6) of the signal having 2<sup>nd</sup> order dynamics.

with the trajectories for  $u$  (blue dotted) and  $\omega$  (green dotted) are shown in the graph below.



**Fig. 4.** Simulated trajectory of a complex signal of a sixth order (high frequency trace), with 2 variable inputs:  $u$  (top trace) and  $\omega$  (bottom).

### 3. KNOWLEDGE DISCOVERY ALGORITHMS

There are several algorithms for estimating the parameters of signal generating models, see ref 1. A reduced order model that can be used to describe the oscillatory behaviour of systems forms an initial value problem of the type:

$$\omega^{-2}.x''+2.\zeta . \omega^{-1}.x'+x=u \quad x(0)=x_0, \quad x'(0)=x'_0$$

Where  $\omega$  is the natural frequency,  $\zeta$  is the damping ratio,  $u$  is the input and  $x$  is the output of the system. The three parameters may be constants, variables or variable with dynamical behaviour.

#### 3.1 Models Assuming Constant Parameters From Single Point Data

Consider using the 1<sup>st</sup> to 4<sup>th</sup> time derivatives at a single point. Given the second order system:

$$\omega^{-2} x'' + 2. \zeta.\omega^{-1}.x' + x = u \quad (1)$$

Differentiate with respect to t:

$$\omega^{-2} x''' + 2. \zeta.\omega^{-1}.x'' + x' = 0 \quad (2)$$

Divide by  $x''$ :

$$\omega^{-2} x'''/ x'' + 2. \zeta.\omega^{-1} + x'/ x'' = 0 \quad (3)$$

Differentiate with respect to t again to give:

$$\omega^{-2} \cdot [(x'' \cdot x'''' - x''''^2) / x''^2] + 0 + [(x''^2 - x' \cdot x''') / x''^2] = 0 \quad (4)$$

We get expressions for estimated  $\omega$ , estimated  $\zeta$ , using (2), and estimated u:

$$E\omega^2 = [x'' \cdot x'''' - x''''^2] / [x' \cdot x''' - x''^2] \quad (5)$$

$$E\zeta = -[E\omega^{-2} x''' + x'] / [2 \cdot E\omega^{-1} \cdot x''] \quad (6)$$

$$Eu = E\omega^{-2} \cdot x'' + 2 \cdot E\zeta \cdot E\omega^{-1} \cdot x' + x \quad (7)$$

### 3.2 Models for Time Varying Parameters

We assume that the first time derivative of u to be non 0. For simplicity we still assume that both a and b (the coefficients of  $x''$  and  $x'$  to make symbol manipulation easier) to be constant and hence disappear on first differentiation. The extra information needed for u' to be non zero is extracted from the 5<sup>th</sup> time derivative of the trajectory.

$$a \cdot x'' + b \cdot x' + x = u \quad (1-a)$$

Differentiate wrt to t and assume u' is non zero to give:

$$a \cdot x''' + b \cdot x'' + x' = u' \quad (8)$$

Differentiate again and set  $u'' = 0$  gives:

$$a.x'''' + b.x''' + x'' = 0 \quad (9)$$

Divide Equation 4 by  $x'''$  to isolate  $b$ :

$$a.x''''/x''' + b + x''/x''' = 0 \quad (10)$$

Differentiate again to eliminate  $b$ :

$$a.(x'''''.x'' - x''''^2)/x''''^2 + (x''''^2 - x'' . x''''')/x''''^2 = 0 \quad (11)$$

Re-arranging for  $a$  gives:

$$a = (x'' . x'''' - x''''^2)/(x'''''.x''' - x''''^2) \quad (12)$$

Solve for  $b$  by substituting  $a$  from equ. 12 into equ. 10:

$$b = -x''/x''' - a.x''''/x'''$$

which after substituting for  $a$  and manipulating gives:

$$b = (x'' . x'''' - x''' . x''''')/(x''''^2 - x''' . x''''') \quad (13)$$

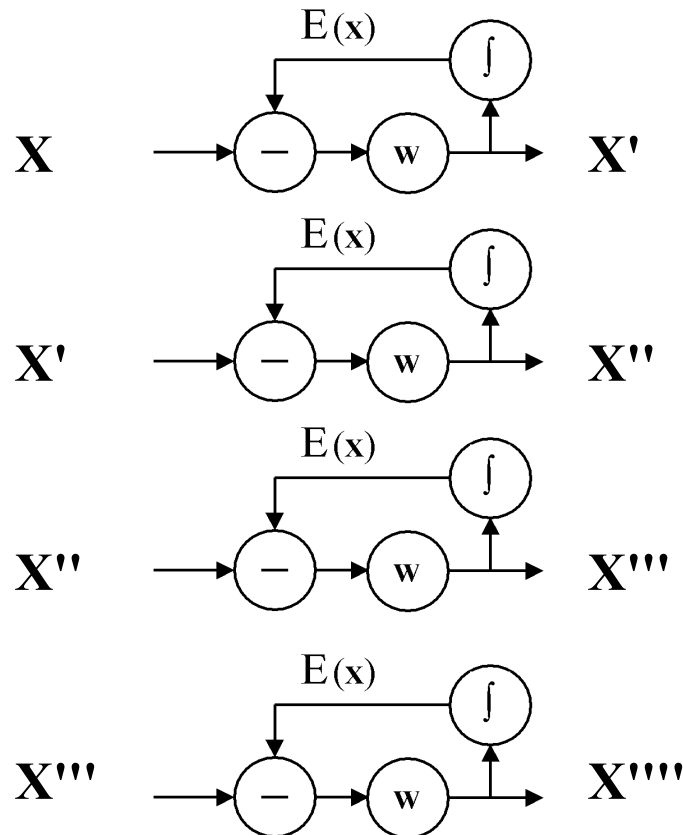
We can now substitute these values for  $a$  and  $b$  into Equation 1 to solve for  $u$ ,

$$u = a.x'' + b.x' + x$$

The essence of the algorithm relies on the accurate estimation of the time derivatives online/real time as the trajectory is tracked This is the subject of the next section.

## 4. ABDUCTION OF TRAJECTORY TIME DERIVATIVES

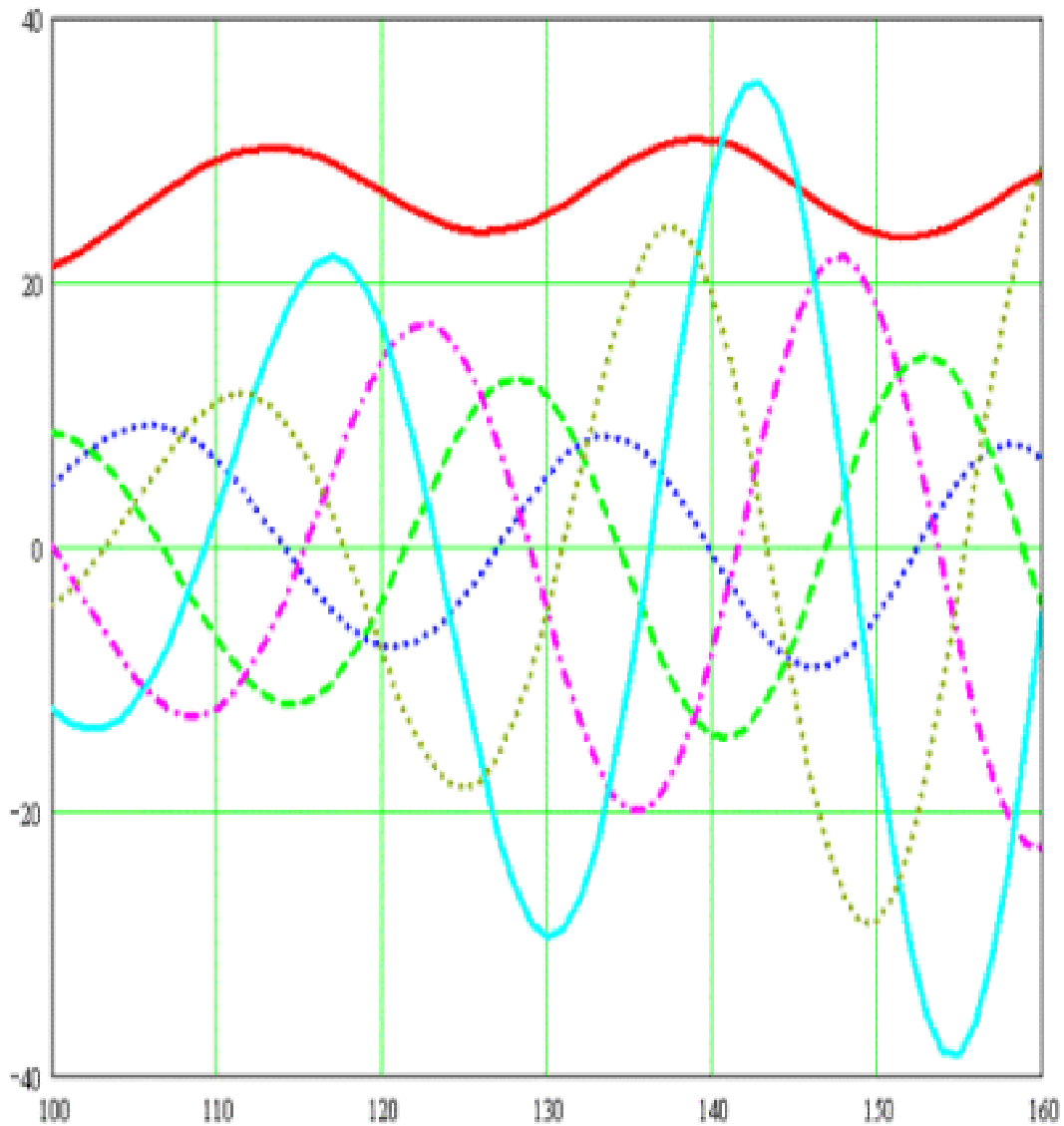
The time derivatives of the trajectory are estimated using a cascade of 1<sup>st</sup> order recurrent networks, Fig. 5. The output of each cell feeds the input to the next one to generate the next higher order time derivative. The output of the system and the cascade of 1<sup>st</sup> order recurrent network filters were simulated using the 4<sup>th</sup> order Runge-Kutta method in Mathcad. The derivatives vector for producing derivatives up to fifth is shown in Fig. 6. Figure 7 shows a typical set of high order time derivatives estimated from the signal.



**Fig. 5.** A 4<sup>th</sup> order recurrent network to estimate 1<sup>st</sup> to 4<sup>th</sup> time derivatives.

$$D(t, x) := \begin{bmatrix}
x_2 \\
x_5 \cdot x_5 \cdot x_3 - 2 \cdot z \cdot x_5 \cdot x_2 - x_5 \cdot x_5 \cdot x_1 \\
x_4 \\
(wu \cdot wu \cdot uu) - 2 \cdot zu \cdot wu \cdot x_4 - wu \cdot wu \cdot x_3 \\
x_6 \\
(ww \cdot ww \cdot uw) - 2 \cdot zw \cdot ww \cdot x_6 - ww \cdot ww \cdot x_5 \\
G \cdot (x_1 - x_7) \\
G1 \cdot [G \cdot (x_1 - x_7) - x_8] \\
G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] \\
G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] - x_{10}] \\
G4 \cdot [G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] - x_{10}] - x_{11}] \\
G5 \cdot [G4 \cdot [G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_7) - x_8] - x_9] - x_{10}] - x_{11}] - x_{12}]
\end{bmatrix}$$

**Fig. 6** The simulation derivative vector, the signal output (top 2 rows), with the input  $u$  modelled in rows 3 and 4, and  $\omega$  in rows 5 and 6; rows 7 to 12 estimate the derivatives as 1st order recurrent nodes.

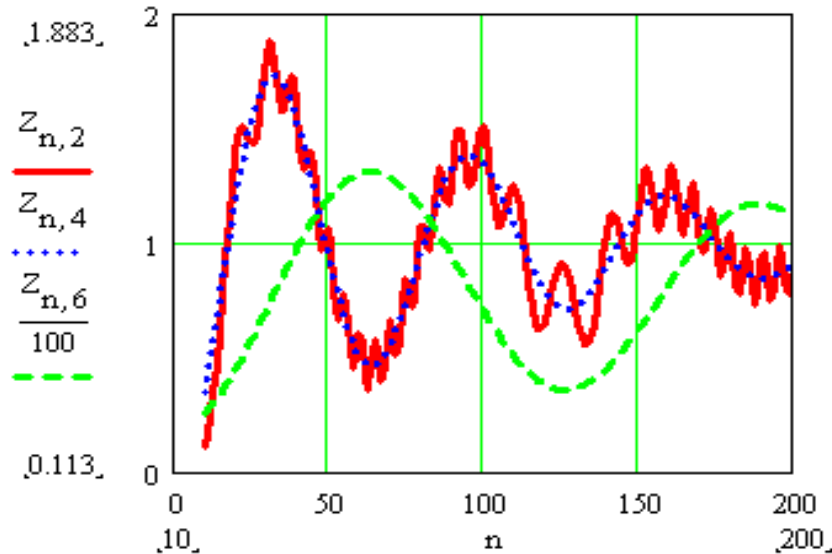


**Fig. 7.** A typical signal trajectory (top, gentle wavy trace) and 5 higher order time derivatives  $x'$  to  $x^{(5)}$  estimated from it.

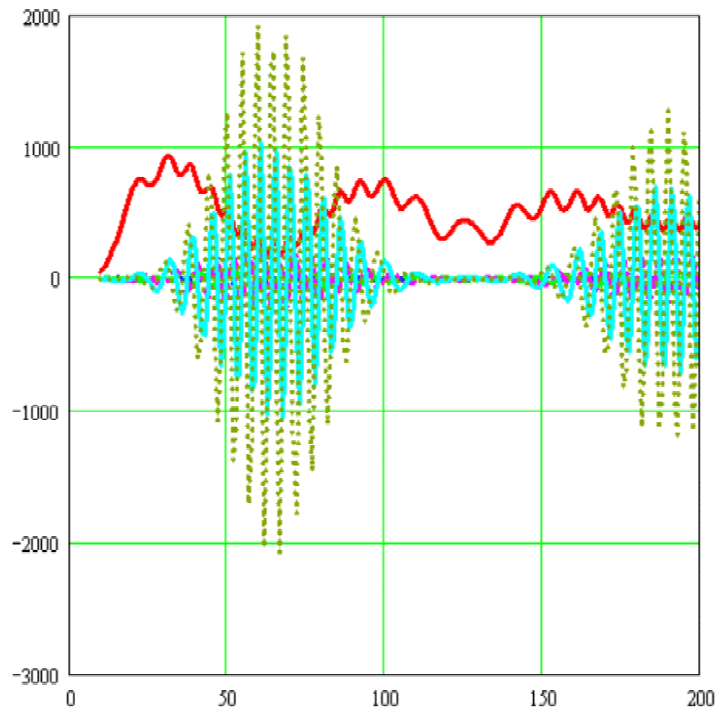
## 5. RESULTS AND DISCUSSION

Mathcad routines were set up to generate the input  $u$  as second order system with its own parameters of natural frequency, damping ratio and input. The input subsystem damping ratio was set to 0.05 to generate an oscillatory behaviour for long enough to test the parameter tracking algorithm thoroughly. The frequency of the input was set to 10 radians per second; the frequency of the main signal, on the other hand, started from 20 and aimed at 80 with a peak of about 130 radians. The derivative generation cascade was increased by one to produce the fifth time derivative. The results are shown in Fig. 8 to 10 below.

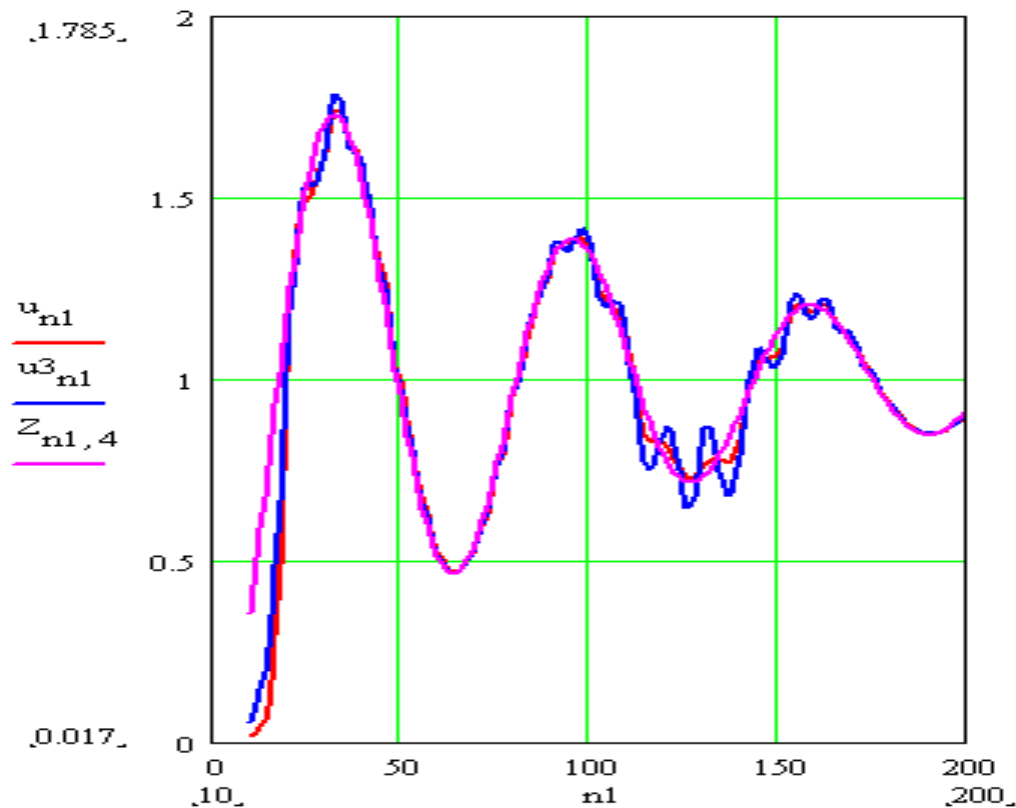
**Fig. 8.** The signal (high frequency trace) following a damped 2<sup>nd</sup> order input (dotted embedded within) with variable natural frequency (low frequency dashed);  $U$ :  $\omega=10$  rad/s,  $\zeta=0.1$ ,  $input=1$  starting from 0;  $\Omega$ :  $\omega=5$  rad/s,  $\zeta=0.05$ ,  $input=80$  starting from 20 rad/s.



**Fig. 9.** The signal (the lone trace at the top) and its estimated high order derivatives (superimposed on each other): high values during periods of high frequency followed by low values for low frequency



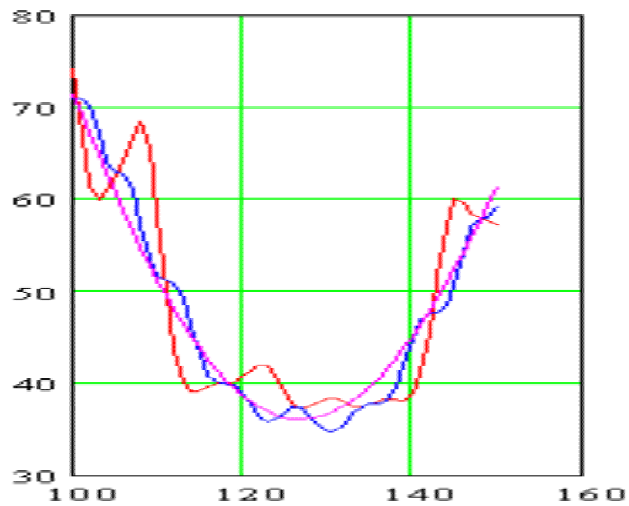
TRACKING TWO PARAMETERS: For a given range of parameters the algorithm worked well, being able to estimate the two input parameters  $u$  and  $\omega$  with their time varying behaviour, i.e. track them while they are changing.



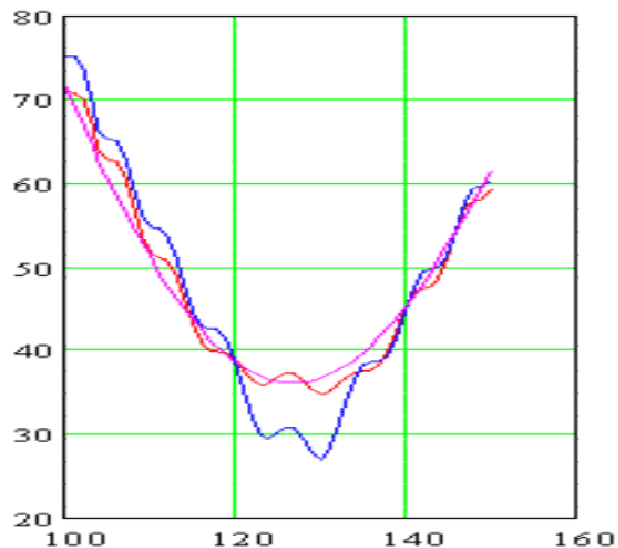
**Fig 10.** Deterioration in the accuracy of estimated input  $u$  during periods of low value high derivatives, particularly prominent for the high order estimation algorithm, e.g. 3<sup>rd</sup> order (very jagged in the 3<sup>rd</sup> quarter of the graph) against 1<sup>st</sup> order (less jagged) and actual value (smooth and starting at a higher value).

The 1<sup>st</sup> order algorithm worked better than the constant one, Fig. 11 shows a comparison of the two algorithms tracking omega.

**Fig. 11.** Estimated Omega assuming constant parameters (very jagged trace) and first order parameters (jagged trace) compared to actual (smooth trace).



**Fig. 12.** Deterioration of accuracy of estimated Omega with higher order algorithms: 3<sup>rd</sup> order estimate (very jagged trace) is worse than 1<sup>st</sup> order estimate (jagged), actual (smooth).



Algorithms of higher order than 1<sup>st</sup> showed marginal improvement but in certain cases showed a deteriorating behaviour, Fig. 12 shows a 3<sup>rd</sup> order algorithm deviating quite markedly from the true trajectory compared to a 1<sup>st</sup> order algorithm. This is likely to be due to an accumulation of errors in higher derivative values used in the former algorithm.

## 6. CONCLUSIONS AND FUTURE WORK

- Estimation models were derived to track the parameters of complicated signals in real time.
- To estimate the values of the parameters, the values of higher order time derivatives had to be estimated first.
- A cascade of 1<sup>st</sup> order recurrent networks were used to for this, followed by non-linear functions to track the parameter values.
- Several of these were tested, the one based on a 1<sup>st</sup> order parameters proved to be the most accurate overall.
- Higher order algorithms gave marginally more accurate results but their accuracy deteriorated under certain conditions.
- Future work will extend the technique to a further layer to track the parameters of a third level input model.
- Application to online stability estimation of large delicate structures will be explored.

**BIOGRAPHY:** David Al-Dabass is professor of Intelligent Systems in the School of Computing & Mathematics, The Nottingham Trent University; he graduated from Imperial College in 1966 with BSc in Electrical Engineering, worked for Redifon Flight Simulation until 1972, completed a PhD in Parallel Processing at Staffordshire University in 1975 and held post-doctoral and advanced research fellowships (76-82) at the Control Systems Centre, UMIST. He joined The Nottingham Trent University in 1983. He is Fellow of the IEE, IMA and BCS and editor-in-chief of the International Journal of Simulation: Systems, Science and Technology; he currently serves as Chair of the UK Simulation Society and Director of European Simulation Multi-conference (ESM) series

for SCS Europe. For more details see his website:  
<http://ducati.doc.ntu.ac.uk/uksim/dad/webpage.htm>

## REFERENCES

- [1] D. Al-Dabass, "Modelling The Complexity Of Concept Dynamics", 46<sup>th</sup> Conference of the Int. Society for the Systems Sciences (ISSS 2002), 2-6 August 2002, Shanghai, pp27-28 ISBN 0-9664183-9-5.
- [2] Al-Dabass, D, Evans D., and Sivayoganathan, K., "Derivative Abduction using a Recurrent Network Architecture for Parameter Tracking Algorithms", *IEEE 2002 Joint Int. Conference on Neural networks, World Congress on Computational Intelligence*, pp1570-1575, Hawaii, May 12-17, 2002.
- [3] D. Al-Dabass, A. Zreiba, D. Evans, S. Sivayoganathan, "Simulation of Three Parameter Estimation, Algorithms for Pattern Recognition Architecture", *UKSIM99 conference, St. Catherine's College, Cambridge, 7-9 April*, pp. 170-176, 1999, <http://ducati.doc.ntu.ac.uk/uksim/papers/moller/dad.doc>.
- [4] A. Zreiba, "MathCad Programs for Parameter Estimation", *Research Report, Dept of Computing, The Nottingham Trent University, Nottingham*, Jan.1999.
- [5] J. D'Azzo and H. Houpis, "Linear Control Systems Analysis and Design", 4<sup>th</sup> ed., *MacGraw Hill series in electrical and computer engineering. Control theory*, 1995.
- [6] W. Press, W. Vetterling, B. Flannery, S. Teukolsky, "Numerical Recipes in C: The Art of Scientific Computing:2<sup>nd</sup> ed.", *Cambridge University Press*, 1992.
- [7] P. Eykhoff, "System Identification Parameter and State Estimation", *John Wiley & sons*, 1974.
- [8] J. Beck and K. Arnold, "Parameter Estimation in Engineering and Science", *John Wiley and sons*, 1977.
- [9] S. Bailey, R. L. Grossman, L. Gu, and D. Hanley, "A Data Intensive Approach to Path Planning and Mode Management for Hybrid Systems," in R. Alur, T. A. Henzinger, and E. Sontag, *Hybrid Systems III, Proceedings of the DIMACS Workshop on Verification and Control of Hybrid Systems*, Springer-Verlag, LNCS 1066, 1996.
- [10] H. Mannila, H. Toivonen, and I. Verkamo, *Discovery of Frequent Episodes in Event Sequences, Data Mining and Knowledge Discovery, Volume 1*, pages 259-289, 1997.
- [11] D. J. Berndt and J. Clifford, *Finding Patterns in Time Series: A Dynamic Programming Approach*, in *Advances in Knowledge Discovery and Data Mining*, edited U. M Fayyad, G. Piatetsky-Shapiro, P. Smyth, and R. Uthurusamy, AAAI Press/MIT Press, pp. 229--248, 1996.