MODELLING THE COMPLEXITY OF CONCEPT DYNAMICS

David Al-Dabass
Dept of Computing and Mathematics
The Nottingham Trent University
Nottingham, NG1 4BU, UK
Email: david.al-dabass@ntu.ac.uk

ABSTRACTS

The notion of concept dynamics is put forward as an aid to categorise a set of concepts according to some behaviour criteria. A set of concepts will change with time under the influence of inputs, - for intelligent entities these inputs may be termed 'senses'. Concepts will manifest their presence through sequences of 'actions' upon their environment. Not all of the concepts are directly measurable from their action sequences, and give rise to the 'observability' condition. Furthermore, senses have only limited ability to change concepts which, in turn, lead to the 'controllability' condition. The behaviour pattern of concepts can be categorised by a set of parameters as well as by senses. Thus suitable parameter estimation algorithms are needed to achieve categorisation.

Keywords: concept models, cognitive dynamics

BIOGRAPHY: David Al-Dabass graduated from Imperial College in 1966 with BSc in Electrical Engineering, worked for Redifon Flight Simulation until 1972, completed a PhD in Parallel Processing at Staffordshire University in 1975 and held post-doctoral and advanced research fellowships (76-82) at the Control Systems Centre, UMIST. He joined The Nottingham Trent University in 1983 as a Principal Lecturer in the Department of Computing. He is Fellow of the IEE, IMA and BCS and editor-in-chief of the International Journal of Simulation: Systems, Science and Technology; he currently serves as Chair of the UK Simulation Society and Director of European Simulation Multi-conference (ESM) series for SCS Europe. For more details see his website: http://ducati.doc.ntu.ac.uk/uksim/dad/webpage.htm

INTRODUCTION

Work with artificial intelligent systems, such as those devised for

- written text recognition (Ren & Al-Dabass 2001) and
- computer game playing (Cant, Churchill & Al-Dabass 2001) lead to the consideration of more
- general conceptual structures in an attempt to
- model higher cognitive processes performed by biological intelligent systems (Goertzel 1994, 1997; Nagel 1992; Penrose 1994; Schank et al 1973).

In this paper an

- analytical approach is adopted
- to model the complexity of concept dynamics.
- Concepts may be modelled as
- mathematical variables, continuous or discrete (Al-Dabass 2001).
- A single concept may be represented by a scalar or a vector variable depending on

- the level of abstraction being considered.

Concepts vary in time in response to input and can be seen to have dynamics.

Concepts Possess Inertia

- It takes a finite time duration to change a concept value and thus it can be said to have inertia.

- A concept subjected to an input will not acquire its new value immediately but evolve gradually.

- Three patterns of behaviour may result:

    - the concept will reach its new value without overshooting it, or

    - it will overshoot initially and then undershoot and overshoot several times before eventually reaching it new value, or

    - the overshoot and undershoot become more divergent and the concept never reaches its intended new value.

Concepts Exhibit Oscillatory Behaviour

- This is an extension to the notion of 'inertia',-
- a concept may embody within its semantics structure multiple 'energy storage facilities' that,
- under the right conditions, such as low damping,
- can operate sufficiently out of phase to cause
- semantic energy to flow back and forth in a similar way that a
- liquid flows between two interconnected reservoirs.

Concept Vector:

Let X be an n-vector of variables that represent the concept.

Senses Vector:

- Let U be an r-vector of variables that represent all inputs to the 'concept system';

- for an intelligent entity (silicon or carbon/biological) these input variables include:

    - sight,
    - hearing,
    - touch and
    - smell as well as

- deeper level messages modulated upon them.

Senses Matrix:

Let the r-vector U influence the Concept rate of change by a time varying n x r matrix C(t).

Actions Vector:

- Let Z be an m-vector of variables that results in
- mapping the concepts onto the environment space.
- No distinction is made here between
  - real physical space and
  - virtual space within a suitable domain such as:
    - software, seen as silicon-based 'mind', or
    - conventional minds as in carbon-biological systems
- inhabited by:
  - the entity generating the concepts and
  - at least one 'other' entity monitoring it.

Action Matrix:

- Let H(t) be an m x n matrix that maps the Concepts n-vector X onto the Actions m-vector Z.

- Therefore the mapping between measurable actions and concepts, i.e.

    - **'Actions as derived from Concepts' equation**,

  is:

- Z(t) = H(t).X(t) Equ. 1

Concepts Dynamical Equation:

Within a dynamical systems context, the concepts vector X evolves in time according to:

- ***the values of its present state,***
- ***input from senses, and***
- ***random disturbance.***

For simplicity of the current treatment let the random disturbance contribution be negligible. This, however, does by no means imply that disturbances are not important, on the contrary their profound significance is best treated in a separate paper.

Rate of Change of Concepts :

- Let X' be the rate of change of concepts such that it is directly influenced by
  - current concept value, plus
  - input from Senses.

- Therefore the **concept dynamics equation** is:

- **$X'(t) = F(t).X(t) + C(t).U(t)$ Equ. 2**

- Where F(t) is n x n time varying matrix which determines how the rate of change of each concept variable is influenced by all the current values of the other concept variables;

- and C(t) is the senses matrix defined above.

- Thus the concepts dynamics are described by a system of first order linear differential equations.

OBSERVABILITY AND CONTROLLABILITY

- Two fundamental ideas involved in modelling the dynamical behaviour of concepts which are related to the
- way concepts are changed by input from senses on the one hand and
- the way they influence actions taken by the entity possessing them on the other are:

  1. Under what conditions can concepts be estimated from measurements of actions.
  2. Under what conditions can concepts be influenced by input received through the senses.

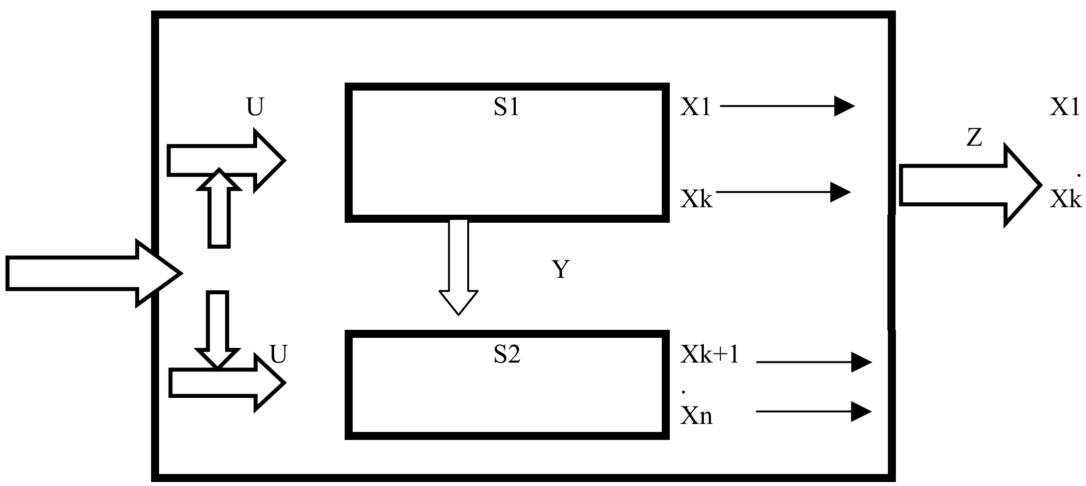

Figure 1: Controllable but unobservable Concept system.

Observability

Consider the continuous linear model formulated above:

$$X'(t) = F(t).X(t) + C(t).U(t)$$

$$Z(t) = H(t).X(t)$$

- **Consider a concept dynamical system S with concept vector X, senses vector U and actions vector Z, refer to Figure 1.**

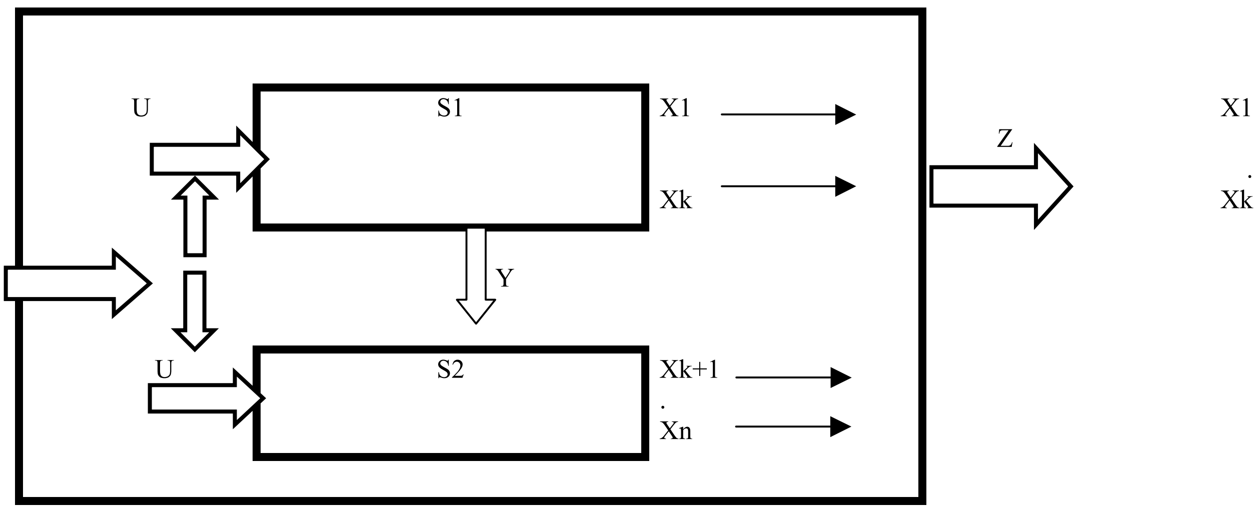

Figure 1: Controllable but unobservable Concept system.

- **Assume that a subset of the concepts, say Y, forms a subsystem S2 such that they do not influence the other concepts X1 - Xk.**

- **Furthermore, the overall systems Actions are derived from X1 - Xk concepts only.**

- **It is then clear that no matter how many sequences of Z are measured, it will not be possible to determine Xk+1 - Xn.**

- **Such a system is clearly controllable but not observable.**

- Consider now the concept system shown in Figure 2.

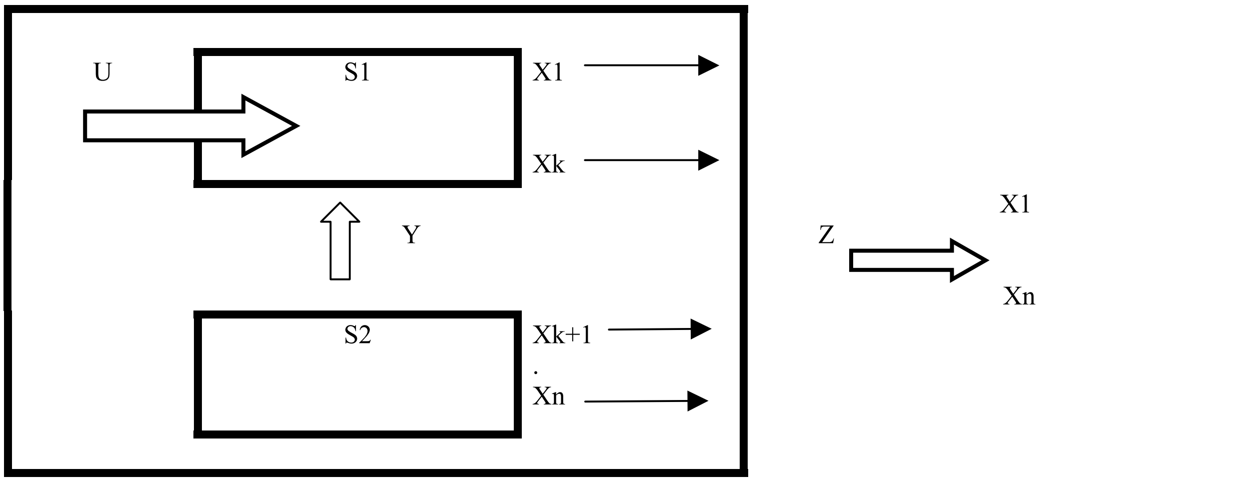

Figure 2: Uncontrollable but observable Concept system.

- The senses now have no influence on the subsystem S2 whether directly or indirectly through the other subsystem (S1) concepts, despite the fact that all concepts are available for measurement.

- This is therefore a completely observable but uncontrollable concept system.

Loosely stated,

- **an Observable system is one where measurement of the Actions is sufficient to determine the Concepts behind the Action; i.e.**

- **when a suitable 'observer' or state re-constructer, can be formed to estimate the values of the concepts vector from the Actions trajectories.**

- **The implication, of course, is that the 'system' can become unobservable when the values of the system matrices F and Z change to give rise to the configuration shown in Figure 1.**

More rigorously, the following is true.

Definition: the system (or any intelligent entity) is observable if the value of its Concepts vector at some time instant t0 X(t0) can be determined from its Actions trajectories Z(t), $t0 \le t \le t1$, for some finite t1. If this is true for any t0, the entity's Concepts are completely observable.

Observability Matrix: is given in terms of Concept Transition matrix CT and Action mapping matrix H as:

$$\mathbf{OM(t0,t1)} = \int_{t0}^{t1} \mathbf{CT''(t,t0)\ .\ H''(t)\ .\ H(t)\ .\ CT(t,t0)\ .\ dt} \qquad \textbf{Equ. 3}$$

Where CT" indicates transpose of CT etc.

Observability Condition: the monitored concept system (entity) as modelled by the linear system F and H is completely observable if and only if the symmetric n x n matrix OM is positive definite for some finite time t1 > t0.

Constant Parameter System: If the matrices F and H are constant, the observability condition simplifies such that the concept system is completely observable if and only if the n x mn matrix

$$\mathbf{H'',\ F''H'',\ \ldots,\ (F'')^{n-1}\ H''} \qquad \textbf{Equ. 4}$$

has rank n.

Controllability

Consider the Concept system given by:

$X' = F(t).X + C(t).U(t)$

For $t \geq t0$, where $X(t0)$ is known but $U(t)$ is not specified.

- The controllability problem involves transferring the Concept values from $X(t0)$ to some required terminal state $X(t1) = X1$ where t1 is finite.
- By suitable change of co-ordinates, the problem becomes that of transferring from some $X(t0)$ to the origin (now given as 0 or $X(t1)$) in a finite time.

Definition:

The concept system given above is controllable at time t0 if there exists an input function (through its senses) $U(t)$ depending on $X(t0)$ and defined over some finite interval
$t0 \leq t \leq t1$ for which $X(t1) = 0$. If this is true for all $X(t0)$ and t0, the concept system is completely controllable.

Controllability Matrix:

is given in terms of the Concept transition matrix CT and the Senses matrix C as:

$$CM(t0,t1) = \int_{t0}^{t1} CT(t0,t1) \,.\, C(t) \,.\, C''(t) \,.\, CT''(t0,t1) \,.\, dt \qquad \text{Equ. 6}$$

Where CT" indicates transpose of CT.

Controllability Condition:

the monitored concept system (entity) as modelled by the linear system F and C is completely controllable if and only if the symmetric n x n matrix CM is positive definite for some finite time t1 > t0.

Constant Parameter System:

If the matrices F and C are constant, the controllability condition simplifies such that the system is completely controllable if and only if the n x nr matrix

$$\mathbf{C, FC, \ldots, F^{n-1} C} \qquad \textbf{Equ. 7}$$

has rank n.

PARAMETER ESTIMATION FOR CONCEPT CATEGORISATION

- Concepts dynamics are defined by differential equations and the matrices F, C and H.
- The numerical values of these matrices determine the behaviour pattern of the concepts.
- Conversely, monitoring the actions generated by concepts should enable us to determine not only the values of the concepts but the exact values of the parameters that determine the nature of their behaviour.
- Consider a concept modelled as a second order dynamical system, the concept state space being its value x and its time derivative x'.
- To simplify the treatment, and without loss of generality, let the concept system be driven by a single sense variable u and generate a single action represented by the concept value itself, i.e. z=x.
- Furthermore, reduce the resulting 2 x 2 system matrix F to represent the concept dynamics in terms of its natural frequency omega and damping coefficient zeta.
- This results in the following well known second order differential equation:

$$\omega^{-2}\, x'' + 2.\, \zeta.\omega^{-1}.x' + x = u \qquad \textbf{Equ. 8}$$

Categorisation of Concepts

- Let the above equation be the model to represent any concept and its dynamics.

- Therefore categorisation reduces to finding the 3 parameters, u, omega and zeta, that completely describe the behaviour pattern of the concept.

- The process of identifying a particular concept thus reduces to monitoring its trajectory and determining these 3 parameters from it.

- Comparison between concepts reduces to comparing the corresponding values of these parameters derived from the behaviour trajectory of the concepts.

- Several algorithms are available for estimating parameters from trajectories depending on
    - the number of points on the trajectory,
    - the number of high order time derivatives available at each point on the trajectory an
    - on the assumption whether the parameters are constant or time varying.

- For the purpose of this illustration it suffices to consider single point algorithms.

- These are classified according to the order of the parameter variation used in the derivation, i.e.
  - constant,
  - first order polynomial (constant u' but u''=0),
  - 2^{nd} order polynomial (constant u'' but u'''=0) etc.
- They will be given here without proof, the interested reader may refer to (Al-Dabass 2002).

Constant Parameters

Consider using the 1^{st} to 4^{th} time derivatives at a single point of the action trajectory generated by the Concept system. Given the second order system:

$$\omega^{-2}\, x'' + 2.\,\zeta.\omega^{-1}.x' + x = u \qquad \text{Equ. 9}$$

We get expressions for estimated ω, estimated ζ, and estimated u:

$$E\omega^{2} = [x''.\, x'''' - x'''^{2}] \,/\, [x'.\, x''' - x''^{2}] \qquad \text{Equ. 10}$$
$$E\zeta = -[E\omega^{-2}\, x''' + x'] \,/\, [2.\, E\omega^{-1}.x'']$$
$$Eu = E\omega^{-2}.\, x'' + 2.\, E\zeta.\, E\omega^{-1}\, .\, x' + x$$

First Order Parameters

Let the first time derivative of u to be non zero. For simplicity assume that both a and b (the coefficients of x'' and x' to make symbol manipulation easier) to be constant and hence disappear on first differentiation. The extra information needed for u' to be non zero is extracted from the 5th time derivative of the trajectory.

$$a.x'' + b.x' + x = u \qquad \text{Equ. 11}$$

By successive differentiation wrt to t, assuming u' is non zero (but u'' and higher time derivatives = 0) and re-arranging for a gives:

$$a = (x'' . x'''' - x'''^{2})/(x''''' . x''' - x''''^{2}) \qquad \text{Equ. 12}$$
$$b = (x''.x''''' - x'''.x'''')/(x'''''^{2} - x''' . x''''')$$
$$u = a.x'' + b.x' + x$$

Simulation Model

Simulation was carried out using Mathcad; the simulation derivative vector is shown in Figure 3.

$$D(t,x) := \begin{bmatrix} x_2 \\ w \cdot w \cdot x_4 - 2 \cdot z \cdot w \cdot x_2 - w \cdot w \cdot x_1 \\ 0 \\ x_5 \\ (wu \cdot wu \cdot uu) - 2 \cdot zu \cdot wu \cdot x_5 - wu \cdot wu \cdot x_4 \\ G \cdot (x_1 - x_6) \\ G1 \cdot [G \cdot (x_1 - x_6) - x_7] \\ G2 \cdot [G1 \cdot [G \cdot (x_1 - x_6) - x_7] - x_8] \\ G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_6) - x_7] - x_8] - x_9] \\ G4 \cdot [G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_6) - x_7] - x_8] - x_9] - x_{10}] \\ G5 \cdot [G4 \cdot [G3 \cdot [G2 \cdot [G1 \cdot [G \cdot (x_1 - x_6) - x_7] - x_8] - x_9] - x_{10}] - x_{11}] \end{bmatrix}$$

Figure 3: rows 1 and 2 from the top show the Concept 2nd order system; row 3 is not used; rows 4 and 5 simulate the input Sense parameter as a second order subsystem; rows 6 to 11 produce estimated x (Action) and its time derivatives x', x'', x''', x'''', x'''''

RESULTS AND DISCUSSION

- Mathcad routines were set up to generate
- the Sense input u as second order system with its own parameters of
  - natural frequency,
  - damping ratio and
  - input.
- The Sense subsystem damping ratio was set to 0.05 to generate an oscillatory behaviour for long enough to test the Sense tracking algorithm thoroughly.
- The frequency of the input was set to 16 radians per second, one quarter of the frequency of the Concept natural frequency.
- The derivative generation cascade was increased by one to produce the fifth time derivative.
- The results are shown in Figure 4 below.

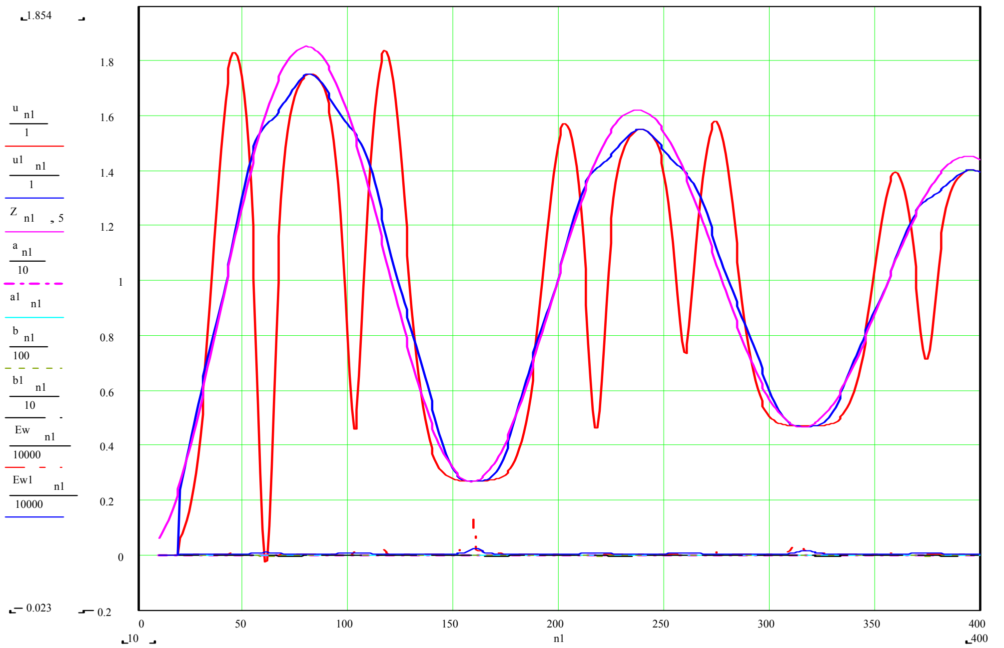

Figure 4: Determining a Sense input using a trajectory of the concept Action.

- The Sense input is shown in pink, which gives approximately 2 and half cycles over a period of one second as expected, i.e. 16 radians/s = 2.546 Hertz.

- The red trace shows the results from the constant u derivation algorithm which is failing completely to track the input.

- The blue trace shows the result of the second algorithm, which is managing to track the input much more closely. It is clear that tracking remains stable.

- It is interesting to note that the first algorithm

    - while completely failing to track the upper half of the Sense input trajectory it seems

    - to track it well during its lower half but not as well as the second algorithm.

CONCLUSIONS

This paper attempted

- To model concepts as vector variables whose time derivatives are related to concepts current values and input from Senses.
- Concepts generate Actions in their environment.
- These actions are measured by other entities to enable them to estimate the Concept values.
- The Sense inputs and the system parameters that determine the behaviour pattern of the concepts and actions.
- The notions of observable and controllable concept systems were introduced and their conditions established.
- Categorisation of concepts based on similarity in the parameter values that determine the nature of their behaviour was put forward.
- Suitable estimation algorithms were investigated to illustrate the procedure of extracting these parameters from the action trajectories of the concept system.
- Current work is extending these ideas into causality in concept systems.

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