A DISCRETE-TIME PERFORMANCE MODEL FOR CONGESTION CONTROL BASED ON RANDOM EARLY DETECTION USING QUEUE THRESHOLDS

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Abstract: In this paper, we present a discrete-time queueing model for the performance evaluation of congestion control based on Random Early Detection (RED) using queue thresholds. The analytical model for a discrete-time finite queue incorporates two thresholds L_1 and L_2 ($L_2>L_1$). Before the number of packets in the system reaches the first threshold L_1, the queue operates normally with arrival rate α_1. When the number of packets in the system is between L_1 and L_2, packets are dropped probabilistically with the drop probability increasing linearly with system contents. Beyond L_2, the drop probability then remains constant until the number of packets reaches the system capacity. The system can be potentially used as an analytical model for congestion control based on RED. For an independent Bernoulli stream, the mean packet delay W, probability of packet loss P_L and throughput S have been found as functions of the thresholds and maximum drop probability. The results clearly demonstrate how different threshold settings can provide different tradeoffs between loss probability and delay to suit different service requirements.

keywords: Queueing Theory, Queue Thresholds, Quality of Service (QoS), Congestion Control

1. INTRODUCTION

With the rapid development of the Internet, the control of congestion has become one of the most critical issues in present networks to accommodate the increasingly diverse range of services and types of traffic [1]. Congestion control to enable different types of Internet traffic to satisfy specified Quality of Service (QoS) constraints is becoming significantly important. Many systems in network environments require the queue to be monitored for impending congestion before it happens [2].

The traditional technique for implementation of router queue management is to set a maximum length (in terms of packets) for each queue, accept packets for the queue until the buffer is full, then reject subsequent incoming packets until the queue length decreases because a packet from the queue has been transmitted. This technique is known as "tail drop", since the packet that arrived most recently (i.e., the one on the tail of the queue) is dropped when the queue becomes full. This method has been used for several years in the Internet with some success, but it has two important drawbacks 'Lock-Out' and 'Full Queues' [3]. In order to solve the problems, some active queue management (AQM) mechanisms have been proposed and implemented to manage the queue lengths, reduce end-to-end latency, reduce packet dropping, and avoid lock-out phenomena so that the control of congestion can be achieved by the use of appropriate buffer management schemes. These mechanisms include random early detection (RED) [4, 12], random early marking (REM) [5, 6], a virtual queue based on scheme where the virtual queue is adaptive [7, 8, 9] and a proportional integral controller mechanism [10], among others. Of these schemes to implement AQM, the RED mechanism is the one recommended by the Internet Society in [3]. RED depends on setting thresholds in the queue and our research looks at this in a simplified way where we have two thresholds L_1 and L_2 which define changes in the packet dropping probability as described in the next section.

2. AN ANALYTICAL MODEL FOR A DISCRETE-TIME FINITE CAPACITY QUEUE WITH TWO THRESHOLDS (L_1 and L_2)

In this queueing system, we will assume that a departure always takes place before an arrival in any unit time (slot). Arrivals form an independent Bernoulli process, with $a_n \in \{0,1\}$, n=1,2,3,…., and there is a finite waiting room of L_2+N packets, including any in service, with two thresholds L_1 and

L_2 (c.f., Fig.1). The queueing discipline is first-come first-served. [11]

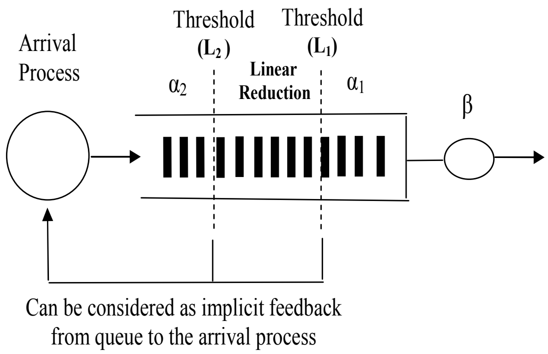

FIGURE 1. Single buffer with two thresholds (L_1 and L_2)

Let the probability of an arrival in a slot be α_1 before the number of packets in the system reaches the first threshold L_1, the probability of an arrival in a slot be reduced to α_2 after the number of packets in the system reaches the second threshold L_2, and the probability of a departure in a slot be β. When the number of packets in the system is between the first threshold and the second threshold, the arrival rate (probability) will be linearly reduced with some probability which is the function of α_1, α_2 and the two thresholds. So the dropping probability increases linearly from 0 to the maximum $1-\alpha_2/\alpha_1$. This can be considered as implicit feedback from queue to the arrival process in that dropping packets reduces the effective arrival rate into the queue from α_1 to α_1- α_2 with a linear reduction. We assume that the queueing system is in equilibrium. The state transition diagram is shown in Fig. 2, and the queue length process is a Markov chain with a finite state space {0, 1, …, L_2+N}.

As shown in Fig. 2, the arrival rate is α_1 in part I and α_2 in part III, which are all independent. However in part II (between two thresholds), the arrival rate depends on the state, that means each arrival rate is different with each state and will be linearly reduced by dropping packets. We assume that $\alpha_1 \neq \beta$, $\alpha_2 \neq \beta$ ($\alpha_1>\alpha_2$) and the final state L_2+N is the full buffer situation.

To find the equilibrium probability, first the transition probabilities of arrivals and departures from state L_1 to state L_2-1 can be defined as:

$$\lambda_k = \alpha_k(1-\beta), \mu_k = \beta(1-\alpha_k), \quad L_1 \leq k \leq L_2 - 1 \tag{2.1}$$

where

$$\alpha_k = \alpha_1 - (k - L_1 + 1)\frac{\alpha_1 - \alpha_2}{L_2 - L_1 + 1}, \quad L_1 \leq k \leq L_2 - 1 \tag{2.2}$$

And the transition probabilities of arrivals and departures in part I and III can also be defined as:

$$\lambda_0 = \alpha_1, \ \lambda_1 = \alpha_1(1-\beta), \ \mu_1 = \beta(1-\alpha_1)$$

$$\lambda_2 = \alpha_2(1-\beta), \ \mu_2 = \beta(1-\alpha_2)$$

The balance equations of the discrete-time finite queue with two thresholds L_1 and L_2 ($L_2>L_1$) can be expressed as follows:

$$\pi_0\lambda_0 = \pi_1\mu_1 \tag{2.3}$$

$$\pi_i\lambda_1 = \pi_{i+1}\mu_1, \qquad 1 \leq i \leq L_1 - 2 \tag{2.4}$$

$$\pi_{L_1-1}\lambda_1 = \pi_{L_1}\mu_{L_1} \tag{2.5}$$

$$\pi_i\lambda_i = \pi_{i+1}\mu_{i+1}, \qquad L_1 \leq i \leq L_2 - 2 \tag{2.6}$$

$$\pi_{L_2-1}\lambda_{L_2-1} = \pi_{L_{21}}\mu_2 \tag{2.7}$$

$$\pi_i\lambda_2 = \pi_{i+1}\mu_2, \qquad L_2 \leq i \leq L_2 + N - 1 \tag{2.8}$$

Solving these equations recursively, and involving $\rho_1 = \frac{\lambda_1}{\mu_1}$ and $\rho_2 = \frac{\lambda_2}{\mu_2}$, after the equilibrium probability can be expressed in terms of π_0, we use the normalising equations $\sum_{i=0}^{L_2+N}\pi_i = 1$, then π_0 can be obtained as following:

$$\pi_0 = \left[\frac{\lambda_1(1-\rho_1) + \lambda_0(\rho_1 - \rho_1^{L_1})}{\lambda_1(1-\rho_1)} + \lambda_0\rho_1^{L_1}\sum_{i=L_1}^{L_2-1}\prod_{k=L_1}^{i-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{1}{\mu_i} + \frac{\lambda_0\rho_1^{L_1}\left(1-\rho_2^{N+1}\right)}{1-\rho_2}\prod_{k=L_1}^{L_2-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{1}{\mu_2}\right]^{-1} \tag{2.9}$$

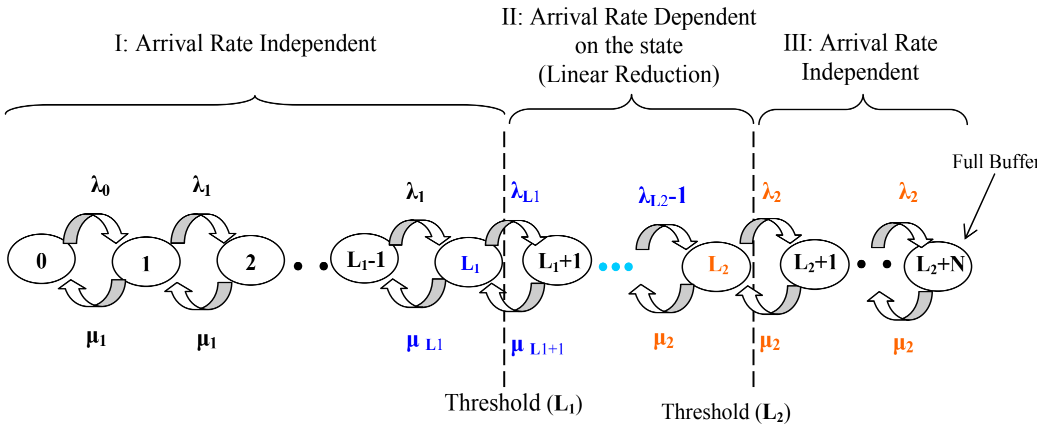

FIGURE 2. State Transition Diagram for the Discrete-Time Finite Queue with two Thresholds (L_1 and L_2)

The idea is to find the generating function of the queue length process for this finite queue which is given by:

$$p(z) = \sum_{i=0}^{L_2+N} \pi_i z^i \qquad (2.10)$$

Multiplying π_i by z^i, and summing them together we can find $P(z)$. To find the mean waiting time via Little's result, we must first evaluate the mean queue length which can be obtained from the generating function done by the first derivative of $P(z)$ evaluated at $z = 1$:

$$P^{(1)}(1) = \pi_0\left[\frac{\lambda_0\rho_1\left(1-\rho_1^{L_1}-\rho_1^{L_1-1}L_1(1-\rho_1)\right)}{\lambda_1(1-\rho_1)^2} + \lambda_0\rho_1^{L_1-1}\sum_{i=L_1}^{L_2-1}\prod_{k=L_1}^{i-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{i}{\mu_i}\right]$$

$$+\pi_0\left[\lambda_0\rho_1^{L_1-1}\prod_{k=L_1}^{L_2-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{1}{\mu_2}\left(\frac{L_2\left(1-\rho_2-\rho_2^{N+1}+\rho_2^{N+2}\right)+\rho_2\left(1-\rho_2^N-N\rho_2^N(1-\rho_2)\right)}{(1-\rho_2)^2}\right)\right]$$

(2.11)

The delay can be obtained from Little's result for this finite capacity queue as:

W =

$$\frac{\pi_0}{S}\left[\frac{\lambda_0\rho_1\left(1-\rho_1^{L_1}-\rho_1^{L_1-1}L_1(1-\rho_1)\right)}{\lambda_1(1-\rho_1)^2} + \lambda_0\rho_1^{L_1-1}\sum_{i=L_1}^{L_2-1}\prod_{k=L_1}^{i-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{i}{\mu_i}\right]$$

$$+\frac{\pi_0}{S}\left[\lambda_0\rho_1^{L_1-1}\prod_{k=L_1}^{L_2-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{1}{\mu_2}\left(\frac{L_2\left(1-\rho_2-\rho_2^{N+1}+\rho_2^{N+2}\right)+\rho_2\left(1-\rho_2^N-N\rho_2^N(1-\rho_2)\right)}{(1-\rho_2)^2}\right)\right]$$

(2.12)

Where S is the mean throughput of this queue given by the fraction of time the server is busy:

$$S = (1-\pi_0)\times\beta \qquad (2.13)$$

Instead of the throughput, another more important performance measure is the probability of packet loss which is given by:

$P_L =$

$$\frac{1}{\alpha_1}\lambda_0\rho_1^{L_1-1}\pi_0\frac{\alpha_1-\alpha_2}{L_2-L_1+1}\sum_{i=L_1}^{L_2-1}\prod_{k=L_1}^{i-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{i-L_1+1}{\mu_i}$$

$$+\left(\lambda_0\rho_1^{L_1-1}\prod_{k=L_1}^{L_2-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{1}{\mu_2}\pi_0\right)\left(\frac{1-\rho_2^N}{1-\rho_2}\left(1-\frac{\alpha_2}{\alpha_1}\right)+\rho_2^N\left(1-\beta\frac{\alpha_2}{\alpha_1}\right)\right)$$

(2.14)

To check the result of mean queue length for this model we use the result from the balance equations for comparison. The mean waiting time can be found directly from Little's result for this finite capacity queue as

$$W = \frac{\sum_{i=0}^{L_2+N} i\pi_i}{S} \qquad (2.15)$$

After manipulation, this gives:

W =

$$\frac{\pi_0}{S}\left[\frac{\lambda_0\rho_1\left(1-\rho_1^{L_1}-\rho_1^{L_1-1}L_1(1-\rho_1)\right)}{\lambda_1(1-\rho_1)^2} + \lambda_0\rho_1^{L_1-1}\sum_{i=L_1}^{L_2-1}\prod_{k=L_1}^{i-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{i}{\mu_i}\right]$$

$$+\frac{\pi_0}{S}\left[\lambda_0\rho_1^{L_1-1}\prod_{k=L_1}^{L_2-1}\left(\frac{\lambda_k}{\mu_k}\right)\frac{1}{\mu_2}\left(\frac{L_2\left(1-\rho_2-\rho_2^{N+1}+\rho_2^{N+2}\right)+\rho_2\left(1-\rho_2^N-N\rho_2^N(1-\rho_2)\right)}{(1-\rho_2)^2}\right)\right]$$

(2.16)

where S is the mean throughput.

This is exactly the result we obtained in equation (2.12), as we should expect, and thus validates the general results obtained from the generating function.

3. GRAPHICAL RESULTS

Using the functions (2.16) and (2.18) for the delay and probability of packet loss respectively, the graphical results of delay and probability of packet loss against L_2-L_1 are shown in Fig. 3, Fig. 4, Fig. 5 and Fig. 6 separately.

(1) α_1, α_2 fixed, L_2 is variable, results for different values of L_1 are compared

Fig. 3 indicates that the absolute value of mean delay is lower for the lower threshold settings, as to be expected. However, this figure also indicates that the change in mean delay depends only on the distance between the queue thresholds and is independent of the positions of the thresholds in the queue. Fig. 4 shows that a lower probability of packet loss that be achieved by using a high setting for the threshold L_1 and a wide separation of the thresholds, with the probability of packet loss tending to converge to the same value for a very wide thresholds separation.

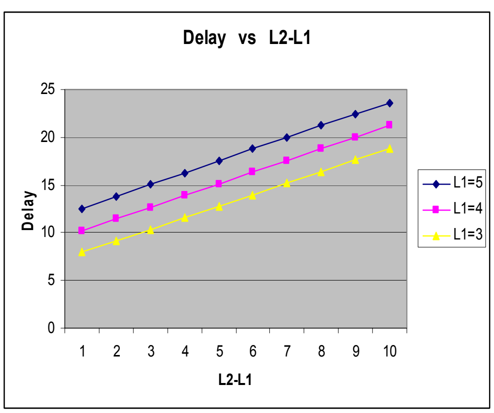

FIGURE 3. Delay vs L_2-L_1

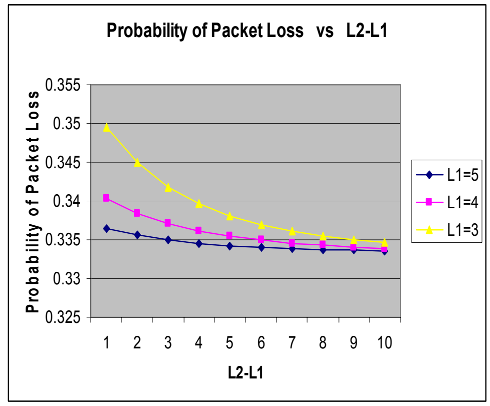

FIGURE 4. Probability of packets loss vs L_2-L_1

(2) α_1, L_1 fixed, L_2 is variable, results for different values of α_2 are compared

Varying the parameter α_2 is the equivalent of varying the maximum drop probability, which is $1-\alpha_2/\alpha_1$. Fig. 5 shows that a higher value of maximum drop probability gives a lower delay for the same threshold separation L_2- L_1. Fig. 6 shows that a lower value of maximum drop probability gives a lower probability of packet loss for the same threshold separation. However, a lower probability of packet loss can be achieved by using a low maximum drop probability and a wide separation of the thresholds, although for a very wide threshold separation the probability of packet loss converges to the same value, irrespective of the maximum drop probability.

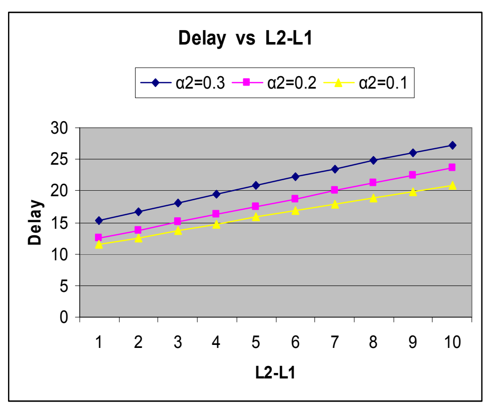

FIGURE 5. Delay vs L_2-L_1

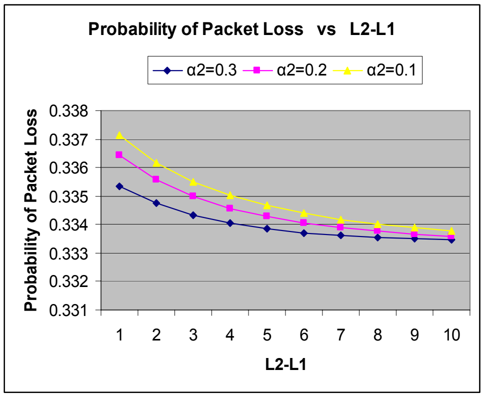

FIGURE 6. Probability of packets loss vs L_2-L_1

4. CONCLUSIONS

The performance model developed and analysed enables the best threshold settings and drop probability to be chosen to suit a given situation; that is, to give an appropriate trade-off between delay and packet loss probability.

Taking the results of Fig. 4, 5, 6 and 7 overall, these suggest that a lower delay for a specific packet loss probability can be obtained by using a high maximum drop probability, a low setting for the threshold L_1 and a narrow separation of the thresholds. A lower probability of packet loss can be achieved by using a low maximum drop probability, a high setting for the threshold L_1 and a wide separation of the thresholds. Settings of these parameters thus can be chosen to suit the type of service required. For example, real-time services require low delay, while data services require low packet loss.

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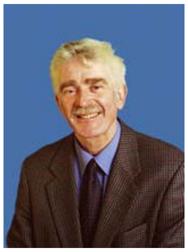
BIOGRAPHIES



Mike E Woodward graduated with a first class honours degree in Electronic and Electrical Engineering from the University of Nottingham in 1967 and received a PhD degree from the same institution in 1971 for research into the decompostion of sequential logic systems. In 1970 he joined the staff of the Department of Electronic and Electrical Engineering at Loughborough University as a lecturer, being promoted to Senior Lecturer in 1980 and Reader in Stochastic Modelling in 1995. He remained at Loughborough unitil 1998 when he was appointed to the Chair in Telecommunications at the University of Bradford where he also became the Director of the Telecommunications Research Centre. He is currently the Head of the Department of Computing at the University of Bradford. His current research interests include queueing networks, telecommunications traffic modelling, quality of service routing and mobile communications systems and he is the author of two books and over one hundred research papers on the above and related topics. He currently holds two EPSRC grants and is supervisor to seven full time and six part time research students. Professor Woodward is a Fellow of the Institute of Mathematics and its Applications (FIMA) and is both a Chartered Mathematician (CMath) and a Chartered Engineer (CEng).

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Lin Guan received the B.Sc degree in computer science from Heilongjiang University, Heilongjiang, China, in 2001. She has studied in the department of computing of University of Bradford since 2000 to finish her first degree through the partnership scheme between two universities. She is currently a PhD student in University of Bradford. Her research interests focus on developing cost effective analytical models for the performance evaluation of congestion control algorithms for Internet traffic and validate them using simulations.